

Kenneth Joseph Arrow (1921-2017)*

Kenneth Arrow, regarded by most as the greatest economist of the second half of the twentieth century, Nobel laureate at the age of 51, passed away on February 21 at the age of 95. His influence spans almost every area of economics. A large number of his contributions, particularly those that were made in 1950s and 1960s, were foundational and resulted in the development of new areas and directions.

Among his many seminal contributions, possibly the greatest was what is now known as the Arrow Impossibility Theorem, which he proved in his Ph.D. dissertation. It resulted in the creation of a new sub-discipline called Social Choice Theory. Arrow Impossibility Theorem says that there does not exist any rule that can aggregate individual rankings into social rankings satisfying some extremely appealing conditions. These conditions are: (i) For every profile of individual rankings, the rule must assign a unique social ranking. That is to say, the rule must work regardless of what individual preferences are. If a rule does not satisfy this condition then it would work in some situations and not work in other situations. (ii) If some alternative x is preferred to another alternative y by everyone in the society then in the social ranking as well x must be preferred over y . (iii) The rule must be non-dictatorial. A person is called a dictator if it is the case that whenever he prefers some alternative x to some other alternative y , so does the society. The non-dictatorship condition requires that there be no dictator. (iv) The rule must be such that the social preferences over any set of alternatives must depend solely on the individual preferences over those alternatives. In particular, the social preferences over any set of alternatives must remain unaffected if individual preferences over those alternatives have not undergone any changes but have undergone changes over other alternatives. These four conditions are known as conditions of unrestricted domain, weak Pareto principle, non-dictatorship, and independence of irrelevant alternatives respectively. Arrow's mono-

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graph Social Choice and Individual Values¹, by now a classic, contains the statement, proof, and exposition of the Impossibility Theorem.

The framework within which Arrow set out his impossibility theorem is very general and can be interpreted in several different ways. Two of these interpretations are particularly important. Under one interpretation the Arrow Impossibility Theorem says that no ‘satisfactory’ voting rule can be designed; the term ‘satisfactory’ having the meaning that the above-mentioned four conditions are fulfilled. Under the second interpretation the Theorem says that there is no satisfactory way of evaluating social welfare on the basis of individual welfares.

The famous paradox of majority voting illustrates the Impossibility Theorem. Consider three candidates x , y , and z . Let the total number of voters be a multiple of three. Let one-third of the voters prefer x to y and y to z (and therefore x to z); one-third of the voters prefer y to z and z to x (and y to x); and one-third of the voters z to x and x to y (and z to y). Then x defeats y in a majority vote as two-thirds of the voters prefer x over y as against only one-third preferring y over x ; y defeats z in a majority vote as two-thirds of the voters prefer y over z and only one-third prefer z over y ; and z defeats x in a majority vote as two-thirds of the voters prefer z over x and only one-third prefer x over z . It is obvious that majority voting satisfies conditions (ii)-(iv); it, however, fails to satisfy condition (i). From Arrow’s Theorem it follows that the problem is not confined to the majority rule only. Every voting rule would violate at least one of the four conditions.

The voting paradox associated with the majority rule was discovered at the end of the 18th century by the French philosopher Marquis de Condorcet. Arrow, however, was not aware of Condorcet’s work and rediscovered the paradox independently. Although the interest in voting rules goes as far back as Roman times², the systematic study of voting rules became possible only after Arrow’s creation of the social choice theoretic framework.

In a personal reminiscence³ Arrow recounted how he came to suspect and then prove

¹Wiley, New York, 1951; Second edition 1963.

²Pliny the Younger in a very interesting letter to a fellow Senator discusses, in the context of voting in the Senate on a particular issue, as to how if they vote in favour of a more preferred alternative and against a less preferred alternative in every vote division then the eventual outcome would be highly undesirable from their perspective; and on the other hand the eventual outcome would be less undesirable if in some vote divisions they vote in favour of a less preferred alternative and against a more preferred alternative (Pliny the Younger Complete Letters, Translated by P.G. Walsh, Oxford University Press, New York, 2006).

³Arrow, K. J., ‘The Origins of the Impossibility Theorem’, in J. K. Lenstra, A. H. G. Rinnooy Kan, A. Schrijver (Eds.), History of Mathematical Programming: A Collection of Personal Reminiscences,

the Impossibility Theorem. In 1949, philosopher Olaf Helmer who was on the staff of the Rand Corporation at that time was finding it difficult to comprehend how one could apply game theory to the case of countries. While it makes sense to assume that individuals have well-defined preferences, what meaning could one possibly ascribe to countries having preferences given that countries have individuals with heterogeneous preferences? When Arrow told Helmer that this problem had been solved by Bergson in terms of utilities and could easily be translated in terms of preferences, he was requested by Helmer to write it up. It was in the process of writing up this exposition that he realized that there was no satisfactory way of arriving at collective preferences on the basis of individual preferences.

Arrow's contributions regarding the market mechanism are also of fundamental importance. Competitive markets are characterized by large numbers of participants who take decisions in ignorance of the decisions of others. The first important question that arises is how one can be sure that such uncoordinated decisions on the part of a large number of participants would not lead to a chaotic situation. In one of the most elegant results of economic theory Arrow in a paper co-authored with Gerard Debreu⁴ showed that under certain conditions such uncoordinated decisions would be compatible with each other in the sense that an equilibrium would exist. (The existence of equilibrium was independently proved by Lionel W. McKenzie.⁵)

One important question that one can ask about any institution or mechanism is whether the outcomes that result under it have some specified desirable property. In Economics Pareto-efficiency is considered a very desirable property. An outcome is Pareto-efficient or Pareto-optimal if it is not possible to make some individuals better-off without making any individual worse-off. There are two fundamental theorems of welfare economics which relate the market with Pareto-efficiency. The first fundamental theorem says that under certain conditions the outcome that results under the market mechanism is Pareto-efficient. The second fundamental theorem says that provided that, by making the required redistributions, the initial endowment is appropriately chosen, then under certain conditions, almost any Pareto-efficient allocation can be obtained through the market mechanism. Although the history of these theorems is long, the first rigorous proofs of them were provided by Arrow.⁶

North-Holland, Amsterdam, 1991.

⁴Existence of Equilibrium for a Competitive Economy, 22 *Econometrica*, 265-290, 1954.

⁵'On Equilibrium in Graham's Model of World Trade and Other Competitive Systems', 22 *Econometrica*, 147-161, 1954.

⁶An Extension of the Basic Theorems of Classical Welfare Economics, in J. Neyman (Ed.), *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability*, University of California, Berkeley and Los Angeles, 507-532, 1951.

The number of areas in which Arrow made fundamental contributions is quite large. Special mention, however, needs to be made of his 1962 paper ‘The Economic Implications of Learning by Doing’⁷ and his 1963 paper ‘Uncertainty and the Welfare Economics of Medical Care’⁸. The former engendered the voluminous literature on endogenous growth more than two decades after its publication; while the latter is credited with giving rise to the sub-discipline of Health Economics.

It is not widely known that Arrow’s first publication was in Meteorology.⁹ His involvement with Meteorology came about as part of his military service. The US Air Force had received a proposal regarding how to use the winds when navigating. Arrow thought that it wasn’t the right question. Rather, the right question was how to get an airplane from one place to another as fast as possible, if you know the winds. This in turn resulted in an interesting mathematical problem. He read the relevant literature, which was mostly in German, made an important innovation and that resulted in the Meteorology paper.

Although Arrow devoted a considerable part of his intellectual life researching markets, he was not a votary of unregulated markets and supported state-intervention for their regulation. He was critical of the viewpoint that equates freedom with the market. He was concerned from an early age about issues like poverty, income inequality, and racial discrimination. In his later years he was deeply involved with issues relating to climate change.

Arrow was a voracious reader. He had learnt to read even before going to school, possibly even before he was four years old. He had a great reputation as someone possessing encyclopaedic knowledge not only of Economics but even of things far removed from the discipline of Economics. There are many anecdotes about his encyclopaedic knowledge and sharpness of intellect that are in circulation. One anecdote that has been around for quite some time, and recounted recently in a write-up that appeared in the New York Times after Arrow’s death, goes something like this: Arrow’s junior colleagues in a good-natured conspiracy to get the better of him decided to read up on the breeding habits of gray whales and gather on a date and at a place where Arrow was sure to come. When Arrow came they all started talking about Turner’s theory explaining how gray whales found the same breeding spot year after year. While leaving Arrow is said to have muttered that he thought that Turner’s theory had been discredited by Spenser. In an interview with Jane Hibbard conducted in 2011 (as part of the Stanford University Oral

⁷29 *Review of Economic Studies*, 155-173.

⁸53 *American Economic Review*, 941-973.

⁹‘On the Use of Winds in Flight Planning’, 6 *Journal of Meteorology*, 150-159, 1949.

History Project) Arrow had this to say regarding this anecdote: ‘In a discussion on the breeding habits of Pacific gray whales, I am supposed to have pointed out that a recent article had refuted the statements others were making. I’ve been assured that I said it. I cannot really remember it; doesn’t even seem credible to me.’ Regardless of whether this anecdote is true or apocryphal, there is no doubt that it very aptly conveys the breadth and depth of knowledge that he possessed.

In contemporary times, among systems, the pre-eminent position is held by democracy in the political domain and by the market in the economic domain. Arrow’s contributions regarding both democracy and the market are of such fundamental importance that it is inconceivable that any worthwhile academic discourse could be possible on either of these two without reference to Arrow’s contributions.

Satish K. Jain
ICSSR National Fellow
Management Development Institute, Gurgaon
satish.k.jain@gmail.com