Decoupled Liability and Efficiency: An Impossibility Theorem*

Satish K. Jain*

Centre for Economic Studies and Planning
School of Social Sciences
Jawaharlal Nehru University
New Delhi 110 067
India

Abstract

A basic feature of tort law is that of coupled liability. The damages awarded to the victim equal liability imposed on the injurer. This feature of tort law is incorporated in the very definition of a liability rule by postulating that the shares of loss borne by the two parties sum to one. In this paper the relationship between this feature of tort law and efficiency is investigated. It is shown that coupled liability is necessary for efficiency, i.e., if a rule is such that it invariably gives rise to efficient outcomes then it must be the case that under it the liability is coupled. In other words, no rule with decoupled liability can be such that it always yields efficient outcomes.

Keywords: Decoupled Liability, Liability Rules, Hybrid Liability Rules, Efficient Rules, Quasi-Efficient Rules, Condition of Negligence Liability.

JEL Classification: K13

* Email: satishkumarjain@gmail.com

*A later version of this paper was published in Review of Law & Economics, Volume 8, Issue 3, 2012, pp. 697-718.

Decoupled Liability and Efficiency: An Impossibility Theorem

Satish K. Jain

A basic feature of tort law is that of coupled liability. When liability is coupled, the damages awarded to the victim equal the liability imposed on the injurer, as is the case under strict liability and negligence. The liability in fact is coupled under all liability rules. An example of a rule with decoupled liability is provided by the rule under which the injurer pays tax equal to the harm and the victim bears her loss. The purpose of this paper is to examine the relationship between the coupled liability feature of tort law and efficiency. The paper rigourously establishes that decoupled liability is inconsistent with efficiency.

For analyzing the relationship between decoupled liability and efficiency we first slightly generalize the standard tort model[1]. In the standard tort model the accident problem is considered within the framework of two-party interaction under the assumption that transaction costs are sufficiently high to preclude a negotiated solution. It is assumed that, in case of occurrence of an accident, the entire loss falls, to begin with, on one party, to be called the victim (plaintiff), the other party being the injurer (defendant). The probability of accident and the amount of loss in case of occurrence of accident are assumed to depend on the levels of care taken by the two parties. Both parties are assumed to be risk-neutral and social goal is taken to be the minimization of total social costs, which are the sum of costs of care taken by the two parties and expected accident loss.

A liability rule determines the proportions x and y, $x + y = 1$, in which the victim and the injurer respectively are to bear the loss in case of accident as a function of whether and by how much the parties' levels of care were below the due care levels. As was noted above, liability is coupled under all liability rules; the reason being that the notion of coupled liability is incorporated in the very definition of a liability rule by the requirement that $x + y = 1$. In order to discuss decoupled liability we introduce the notion of a hybrid liability rule, to be written as h-liability rule in abbreviated form. Like a

[1]The standard tort model is essentially the one first formulated by Brown (1973) and elaborated by Landes and Posner (1987) and Shavell (1987).

liability rule, a h-liability rule also determines the proportions in which the two parties are to bear the loss in case of accident as a function of whether and by how much the parties' levels of care were below the due care levels. But unlike the case of a liability rule there is no requirement that the liability shares of the two parties must sum to one. The sum of the liability shares of the two parties can be any nonnegative number. Thus the notion of a liability rule is a special case of a h-liability rule. The definition of a h-liability rule reduces to that of a liability rule when the sum of the liability shares equals 1.

A h-liability rule is efficient iff it invariably induces both parties to take total social cost minimizing care levels.[2] The main theorem of the paper shows that no h-liability rule with decoupled liability can be such that it invariably induces both parties to take total social cost minimizing care levels. In other words, in the context of h-liability rules, decoupled liability is inconsistent with efficiency. The paper is divided into four sections. The first section spells out the framework of analysis and contains definitions and assumptions. Section 2 discusses the logic of the impossibility theorem. Although no decoupled h-liability rules are efficient, some of them exhibit the interesting property of always yielding the configuration of due care levels as a Nash equilibrium. The characterization of all h-liability rules exhibiting this property is discussed in Section 3. The concluding remarks are contained in the last section. All formal statements and proofs have been relegated to the Appendix.

1 Definitions and Assumptions

We consider accidents resulting from interaction of two parties, assumed to be strangers to each other, in which, to begin with, the entire loss falls on one party to be called the victim (plaintiff). The other party would be referred to as the injurer (defendant). We denote by $c \geq 0$ the cost of care taken by the victim; and by $d \geq 0$ the cost of care taken by the injurer. We assume that c and d are strictly increasing functions of levels of care of the two parties. This of course implies that c and d themselves can be taken as indices of levels of care of the victim and the injurer respectively.

[2]There is an extensive literature on the efficiency of liability rules. Pioneering contributions were made by Coase (1960) and Calabresi (1961). Calabresi (1961, 1970) dealt with the effect of liability rules on parties' behaviour. The efficiency of the rule of negligence was analyzed by Posner (1972). The formal analysis of some of the most important liability rules was first put forward by Brown (1973). He showed that the rule of negligence as well as the rule of strict liability with the defense of contributory negligence induce both the victim and the injurer to take optimal levels of care. Systematic and detailed treatment of liability rules is contained in Shavell (1987) and Landes and Posner (1987). A complete characterization of efficient liability rules has been obtained in Jain and Singh (2002).

Let
$C = \{c \mid c \text{ is the cost of some feasible level of care which can be taken by the victim}\}$;
and
$D = \{d \mid d \text{ is the cost of some feasible level of care which can be taken by the injurer}\}$.
We assume $0 \in C \wedge 0 \in D$. (A1)

Let π denote the probability of occurrence of accident and $H \geq 0$ the loss in case of occurrence of accident. Both π and H will be assumed to be functions of c and d; $\pi = \pi(c, d), H = H(c, d)$. Let $L = \pi H$. L is thus the expected loss due to accident. We assume:

$$(\forall c, c' \in C)(\forall d, d' \in D)[[c > c' \rightarrow L(c, d) \leq L(c', d)] \wedge [d > d' \rightarrow L(c, d) \leq L(c, d')]]. \tag{A2}$$

In other words, it is assumed that a larger expenditure on care by either party, given the expenditure on care by the other party, results in lesser or equal expected accident loss. Total social costs (TSC) are defined to be the sum of cost of care by the victim, cost of care by the injurer, and the expected loss due to accident; $TSC = c + d + L(c, d)$. Let $M = \{(c', d') \in C \times D \mid c' + d' + L(c', d') \text{ is minimum of } \{c + d + L(c, d) \mid c \in C \wedge d \in D\}\}$. Thus M is the set of all costs of care configurations (c', d') which are total social cost minimizing. It will be assumed that:
C, D, π and H are such that M is nonempty. (A3)

In order to characterize a party's level of care as negligent or otherwise a reference point (the due care level) for the party needs to be specified. Let c^* and d^*, where $(c^*, d^*) \in M$, denote the due care levels of the victim and the injurer respectively. We define nonnegligence functions p and q as follows:

$p : C \mapsto [0, 1]$ such that[3]:

$$\begin{aligned} p(c) &= \tfrac{c}{c^*} \quad \text{if } c < c^*; \\ &= 1 \quad \text{if } c \geq c^* \end{aligned}$$

$q : D \mapsto [0, 1]$ such that:

$$\begin{aligned} q(d) &= \tfrac{d}{d^*} \quad \text{if } d < d^*; \\ &= 1 \quad \text{if } d \geq d^*. \end{aligned}$$

In case there is a legally binding due care level for the plaintiff, it would be taken to be identical with c^* figuring in the definition of function p; and in case there is a legally

[3] Let a and b be real numbers such that $a < b$. We use the standard notation to denote: $\{x \mid a \leq x \leq b\}$ by $[a, b]$, $\{x \mid a \leq x < b\}$ by $[a, b)$, $\{x \mid a < x \leq b\}$ by $(a, b]$, and $\{x \mid a < x < b\}$ by (a, b). $\{x \mid a \leq x\}$ and $\{x \mid a < x\}$ will be denoted by $[a, \infty)$ and (a, ∞) respectively.

binding due care level for the defendant, it would be taken to be identical with d^* figuring in the definition of function q. Thus implicitly it is being assumed that the legally binding due care levels are always set appropriately from the point of view of minimizing total social costs.

p and q would be interpreted as proportions of nonnegligence of the victim and the injurer respectively. The victim would be called negligent if $p < 1$ and nonnegligent if $p = 1$. Similarly, the injurer would be called negligent if $q < 1$ and nonnegligent if $q = 1$.

We now proceed to define the notion of a liability rule. From a technical point of view it is desirable to define the notion of a liability rule independently of its applications. The context in which a liability rule can be applied is completely specified if in addition to C, D, π and H we also specify the configuration of due care levels $(c^*, d^*) \in M$. The set of all applications satisfying (A1) - (A3) would be denoted by $\mathcal{A}$.

Formally, a liability rule is a function f from $[0,1]^2$ to $[0,1]^2$, $f : [0,1]^2 \mapsto [0,1]^2$, such that: $f(p,q) = (x,y)$, where $x + y = 1$.

Thus a liability rule is a rule which specifies the proportions in which the two parties are to bear the loss in case of occurrence of accident as a function of proportions of nonnegligence of the two parties. As in case of accident to begin with the entire loss falls on the victim, yH constitutes the payment by the injurer to the victim. The remaining loss of xH is borne by the victim. It is important to note that the net payment by the injurer equals the net payment to the victim.

The notion of a hybrid liability rule, which we introduce now, is more general than that of a liability rule. Let $\overline{s} \geq 0$ be a given constant. Formally, a hybrid liability rule (h-liability rule) is a function f from $[0,1]^2$ to $[0,\infty)^2$, $f : [0,1]^2 \mapsto [0,\infty)^2$, such that: $f(p,q) = (x,y)$, where $x + y = \overline{s}$. Thus a h-liability rule is a rule which specifies the multiples of loss the two parties are to bear in case of occurrence of accident as a function of proportions of nonnegligence of the two parties. If $\overline{s} = 1$ then the definition of a h-liability rule reduces to that of a liability rule.

If $\overline{s} \neq 1$ then in case of harm it must necessarily be the case that the net payment by the injurer is unequal to the the net payment to the victim; as $yH \neq (1-x)H$. $yH - (1-x)H$ represents the net payment to the government as tax. Thus the liability is decoupled iff $\overline{s} \neq 1$; and coupled iff $\overline{s} = 1$.

Let f be a h-liability rule. Consider a particular application of f given by: $< C, D, \pi, H, (c^*, d^*) \in M > \in \mathcal{A}$. If accident takes place and loss of $H(c,d)$ materializes, then $xH(c,d)$ will be borne by the victim and $yH(c,d)$ by the injurer; where $(x,y) = f(p,q) = f[p(c), q(d)]$. The expected costs of the victim and the injurer, to be denoted by EC_1 and EC_2 respectively, therefore are $c + xL(c,d)$ and $d + yL(c,d)$ respectively. Both parties are assumed to prefer smaller expected costs to larger expected costs and be indifferent between alternatives with equal expected costs.

Let $f : [0,1]^2 \mapsto [0,\infty)^2$ be a h-liability rule. f is defined to be efficient for a given application $< C, D, \pi, H, (c^*, d^*) \in M > \in \mathcal{A}$ iff $(\forall(\overline{c}, \overline{d}) \in C \times D)[(\overline{c}, \overline{d})$ is a Nash equilibrium $\rightarrow (\overline{c}, \overline{d}) \in M]$ and $(\exists(\overline{c}, \overline{d}) \in C \times D)[(\overline{c}, \overline{d})$ is a Nash equilibrium]. In other words, a h-liability rule is efficient for a particular application iff (i) every Nash equilibrium $(\overline{c}, \overline{d}) \in C \times D$ is total social cost minimizing, and (ii) there exists at least one Nash equilibrium $(\overline{c}, \overline{d}) \in C \times D$. f is efficient for a class of applications (or f is efficient with respect to a class of applications) iff it is efficient for every application belonging to that class.

Remark 1 *It should be noted that no h-liability rule can be efficient for an application for which (A3) is not satisfied.*

Throughout this paper we denote $f(1,1)$ by (x^*, y^*), i.e., we write $x(1,1)$ as x^* and $y(1,1)$ as y^*. We also denote $L(c^*, d^*)$ by L^*.

The following examples illustrate some of the concepts discussed above.

Example 1

Let liability rule $f : [0,1]^2 \mapsto [0,1]^2$ be defined by:
$(\forall p, q \in [0,1])[[q < 1 \rightarrow x(p,q) = 0 \wedge y(p,q) = 1] \wedge [q = 1 \rightarrow x(p,q) = 1 \wedge y(p,q) = 0]]$.
f is the familiar negligence rule.
We consider the application specified below.
Let C and D be given by:
$C = \{0,1,2\}, D = \{0,1,2\}$.
For $(c,d) \in C \times D$, let $L(c,d)$ be as given in the following array:

		d		
		0	1	2
	0	10	6	5
c	1	6	2	1
	2	5	1	0

We have $M = \{(1,1),(1,2),(2,1),(2,2)\}$.
Let $(c^*, d^*) = (1,1)$.
Under the negligence rule, the following array gives (EC_1, EC_2) for $(c,d) \in C \times D$.

		d		
		0	1	2
	0	$(\underline{0},10)$	$(6,\underline{1})$	$(5,2)$
c	1	$(1,6)$	$(\underline{3},\underline{1})$	$(\underline{2},2)$
	2	$(2,5)$	$(\underline{3},\underline{1})$	$(\underline{2},2)$

Thus $(1,1)$ and $(2,1)$ are the only $(c,d) \in C \times D$ which are Nash equilibria. As both $(1,1)$ and $(2,1)$ belong to M it follows that f is efficient for the application under consideration.

Example 2

Let h-liability rule $f : [0,1]^2 \mapsto [0,\infty)^2$ be defined by:
$(\forall p, q \in [0,1])[f(p,q) = (1,1)]$.
Under f, regardless of the nonnegligence proportions, the victim bears her loss and the injurer pays tax equal to the loss.
We consider the same application as in Example 1.

Under the h-liability rule f, the following array gives (EC_1, EC_2) for $(c,d) \in C \times D$.

		d		
		0	1	2
	0	$(10,10)$	$(6,\underline{7})$	$(5,\underline{7})$
c	1	$(\underline{7},6)$	$(\underline{3},\underline{3})$	$(\underline{2},\underline{3})$
	2	$(\underline{7},5)$	$(\underline{3},\underline{2})$	$(\underline{2},\underline{2})$

$(c,d) \in C \times D$ which are Nash equilibria are: $(1,1)$, $(1,2)$, $(2,1)$, $(2,2)$. As all of these belong to M it follows that f is efficient for the application under consideration.

2 Efficiency of h-Liability Rules

Intuitively, it seems that the rule under which the victim bears her loss and the injurer pays tax equal to the loss inflicted on the victim should always give rise to efficient outcomes as both parties are forced to internalize the entire loss due to accident. In Example 2 this rule did turn out to be efficient for the application considered therein. However, somewhat surprisingly, it turns out that this rule does not always lead to efficient outcomes, as the following example shows:

Example 3

Consider the following application of the h-liability rule defined by: $(\forall p, q \in [0,1])[f(p,q) = (1,1)]$.
$C = D = \{0,1\}$.
For $(c,d) \in C \times D$, let $L(c,d)$ be as given in the following array:

$$\begin{array}{cc|cc} & & & d \\ & & 0 & 1 \\ \hline & 0 & 100 & 90.5 \\ c & & & \\ & 1 & 90 & 89.9 \end{array}$$

We have $M = \{(1,0)\}$. Let $(c^*, d^*) = (1,0)$.
We obtain $(EC_1(c,d), EC_2(c,d))$ for $(c,d) \in C \times D$ as given in the following array:

$$\begin{array}{cc|cc} & & & d \\ & & 0 & 1 \\ \hline & 0 & (100,100) & (\underline{90.5}, \underline{91.5}) \\ c & & & \\ & 1 & (\underline{91}, \underline{90}) & (90.9, 90.9) \end{array}$$

Thus both $(1,0)$ and $(0,1)$ are Nash equilibria.
As $(0,1) \notin M$, it follows that the rule is inefficient for this application.

The reason why this rule does not invariably result in efficient outcomes in spite of both the parties being forced to internalize the entire loss is due to the fact that the liability under the rule is decoupled. In Theorem 1 it is shown that if a h-liability rule is efficient for every application belonging to $\mathcal{A}$ then the liability must be coupled. In other words, a necessary condition for a h-liability rule to be efficient for every application belonging to $\mathcal{A}$ is that the liability be coupled. An equivalent statement would be that a necessary condition for a h-liability rule to be efficient for every application belonging to $\mathcal{A}$ is that it be a liability rule.

The proof of necessity of coupled liability for a h-liability rule to be efficient for every application belonging to $\mathcal{A}$ is given in the Appendix. Here we discuss the arguments which establish the result informally. To begin with, it is shown in Lemma 1 that a necessary condition for a h-liability rule f to be efficient with respect to $\mathcal{A}$ is that the structure of f be such that when one party is negligent and the other is nonnegligent, the liability of the negligent party must be at least equal to the total loss, i.e., the liability share of the negligent party must be greater than or equal to 1. Suppose f is such that when one party is nonnegligent (say the victim) with $p = 1$ and the other party (the injurer) is negligent with $q = q_0 < 1$, the liability assignments are such that $y(1, q_0) < 1$. Consider an application of f belonging to $\mathcal{A}$ such that: (i) TSC-minimizing configuration is unique, $M = \{(c_0, d_0)\}$, $(c_0, d_0) = (c^*, d^*)$, (ii) both c^* and d^* are positive; and if both parties take optimal care then the expected loss is zero, and (iii) $q_0 d^*$ is an element of D. Given that the victim is using c^*; if we consider a shift by the injurer from d^* to $q_0 d^*$ then the increase in expected loss $L(c^*, q_0 d^*) - L(c^*, d^*) = L(c^*, q_0 d^*)$ ($L(c^*, d^*)$ being 0) must be greater than the reduction in cost of care $(1 - q_0)d^*$ as TSC-minimizing configuration is unique. The injurer, however, bears only a part of the increase in expected loss as $y(1, q_0) < 1$. On the other hand, the entire decrease in cost of care accrues to the injurer. Therefore, it follows that one can always find an application such that, in addition to (i)-(iii), $L(c^*, q_0 d^*) > (1 - q_0)d^* > y(1, q_0)L(c^*, q_0 d^*)$. This establishes the existence of an application for which the unique TSC-minimizing configuration is not a Nash equilibrium; and consequently the existence of an application for which the rule is inefficient. This establishes the necessity of the requirement that when one party is negligent and the other is nonnegligent the liability share of the negligent party must be greater than or equal to 1 for efficiency with respect to $\mathcal{A}$. The requirement in turn implies that no h-liability rule with $\overline{s} < 1$ can be efficient with respect to $\mathcal{A}$ (Corollary 1).

The next step in the argument (Lemma 3) shows that if f is efficient with respect to $\mathcal{A}$ then its structure must be such that when both parties take at least due care neither party's liability share exceeds 1. Suppose h-liability rule f is such that $x(1, 1) = x^* > 1$. Consider an application of f belonging to $\mathcal{A}$ such that: (i) TSC-minimizing configuration is unique, $M = \{(c_0, d_0)\}$, $(c_0, d_0) = (c^*, d^*)$, (ii) $L(c^*, d^*) > 0$, and (iii) C contains $c^* + \epsilon$, $\epsilon > 0$. Given that the injurer is using d^*; if we consider a shift by the victim from c^* to

$c^* + \epsilon$ then the decrease in expected loss $L(c^*, d^*) - L(c^* + \epsilon, d^*)$ must be less than the increase in cost of care ϵ, as TSC-minimizing configuration is unique. The entire increase in cost of care falls on the victim; but the reduction in the amount of expected loss borne by the victim is greater than the reduction in expected loss $L(c^*, d^*) - L(c^* + \epsilon, d^*)$ as $x^* > 1$. Therefore, it follows that one can always find an application such that, in addition to (i)-(iii), $x^*[L(c^*, d^*) - L(c^* + \epsilon, d^*)] > \epsilon > L(c^*, d^*) - L(c^* + \epsilon, d^*)$. This establishes the existence of an application for which the unique TSC-minimizing configuration is not a Nash equilibrium; and consequently the existence of an application for which the rule is inefficient. This establishes the necessity of the requirement that when both parties are nonnegligent neither party's liability share must exceed 1 for efficiency with respect to $\mathcal{A}$. The requirement in turn implies that no h-liability rule with $\overline{s} > 2$ can be efficient with respect to $\mathcal{A}$ (Corollary 3).

Thus the set of h-liability rules which are efficient with respect to $\mathcal{A}$ is contained in the set of h-liability rules for which $1 \leq \overline{s} \leq 2$. The next step in the argument shows that if f is such that $1 < \overline{s} \leq 2$ then the rule cannot be efficient with respect to $\mathcal{A}$. It is shown that, if $1 < \overline{s} \leq 2$ then one can find an application belonging to $\mathcal{A}$ in which a configuration where both parties are taking more than due care, though not TSC-minimizing, is a Nash equilibrium under f. The proof of this part is rather complex. But, intuitively the reason why one can always find such an application is that a configuration which does not minimize TSC and where both parties are taking more than due care becomes a Nash equilibrium under f because $\overline{s} > 1$ implies that reduction in the expected liability which takes place for both the parties together is more than the reduction in the total expected loss.

This completes the argument establishing the impossibility of having both decoupled liability and efficiency with respect to $\mathcal{A}$; as liability is decoupled iff $\overline{s} \neq 1$, and coupled iff $\overline{s} = 1$. In view of the impossibility theorem it follows that the set of h-liability rules efficient with respect $\mathcal{A}$ is identical with the set of liability rules efficient with respect $\mathcal{A}$. As a liability rule is efficient with respect to $\mathcal{A}$ iff it satisfies the condition of negligence liability[4], we conclude that a h-liability rule is efficient for every application belonging to $\mathcal{A}$ iff it satisfies the condition of negligence liability.

[4] A liability rule f satisfies the condition of negligence liability iff $[[\forall p \in [0,1)][f(p,1) = (1,0)] \wedge [\forall q \in [0,1)][f(1,q) = (0,1)]]$. Less formally, a liability rule satisfies the condition of negligence liability iff its structure is such that (i) whenever the injurer is nonnegligent and the victim is negligent, the entire loss in case of occurrence of accident is borne by the victim, and (ii) whenever the victim is nonnegligent and the injurer is negligent, the entire loss in case of occurrence of accident is borne by the injurer.
The necessity and sufficiency of negligence liability for efficiency with respect to $\mathcal{A}$ is shown in Jain and Singh (2002).

3 Characterization of Quasi-Efficient h-Liability Rules

Although the rule under which the victim bears her loss and the injurer pays tax equal to the loss inflicted on the victim considered in Examples 2 and 3, like any other rule with decoupled liability, is not efficient with respect to $\mathcal{A}$; it does have an interesting property. Under this rule the configuration of due care levels $(c^*, d^*) \in M$ is always a Nash equilibrium. Let us define a h-liability rule to be quasi-efficient with respect to an application $< C, D, \pi, H, (c^*, d^*) \in M >$ iff $(c^*, d^*) \in M$ is a Nash equilibrium. A h-liability rule is defined to be quasi-efficient with respect to a class of applications iff it is quasi-efficient for every application belonging to that class. A rule being quasi-efficient with respect to an application of course is no guarantee that the outcome corresponding to that application would be efficient. But, unlike the cases where all equilibria are non-TSC minimizing, the possibility of an efficient outcome is not ruled out altogether either. Thus the class of h-liability rules which are quasi-efficient with respect to $\mathcal{A}$ is of some interest. Theorem 3 given in the Appendix completely characterizes the set of h-liability rules which are quasi-efficient with respect to $\mathcal{A}$. It is shown that a h-liability rule f is quasi-efficient with respect to $\mathcal{A}$ iff it satisfies the following two properties: (i) when one party is negligent and the other is nonnegligent the liability share of the negligent party must be greater than or equal to 1, and (ii) when both parties take at least the due care neither party's liability share exceeds 1. It should be noted that these two conditions together, as we have seen earlier, imply that $1 \leq \bar{s} \leq 2$.

While discussing the logic of the impossibility theorem in the previous section it was noted that if (i) is violated or (ii) is violated by f then one can find an application belonging to $\mathcal{A}$ in which $(c^*, d^*) \in M$ is not a Nash equilibrium. Thus both (i) and (ii) are necessary for quasi-efficiency with respect to $\mathcal{A}$. An intuitive explanation of sufficiency of (i) and (ii) for quasi-efficiency with respect to $\mathcal{A}$ is as follows. Suppose the injurer is using d^*. Consider a change by the victim from c^* to some $c > c^*$. Now, the reduction in expected loss $L(c^*, d^*) - L(c, d^*)$ cannot exceed the increase in cost of care $c - c^*$, otherwise (c^*, d^*) could not have been a TSC-minimizing configuration. As $x^* \leq 1$ by condition (ii), it follows that the reduction in the victim's liability $x^*[L(c^*, d^*) - L(c, d^*)]$ must be less than or equal to the increase in victim's expenditure on care $c - c^*$. Thus, a change from c^* to $c > c^*$ cannot be advantageous for the victim. Next consider a change from c^* to some $c < c^*$. The increase in expected loss $L(c, d^*) - L(c^*, d^*)$ must be at least as large as the decrease in cost of care $c^* - c$, otherwise (c^*, d^*) could not have been TSC-minimizing. By condition (i) and (ii), the increase in victim's liability must be at least equal to the increase in expected loss. Consequently a shift from c^* to $c < c^*$ can never be advantageous for the victim. Thus, given the choice of d^* by the injurer, c^* is best for the victim. By an analogous argument one shows that, given the choice of c^* by the victim, d^* is best for the injurer.

4 Concluding Remarks

This paper has analyzed coupled liability, one of the built-in features of tort law; and has found it to be crucial for efficiency. Consequently, an important implication of the analysis of this paper is that the rules with decoupled liability are unlikely to be of much significance within a framework which is sufficiently general and in which efficiency is accorded a preeminent position. In limited contexts, however, the study of decoupled rules can be useful for at least two reasons. As a rule is efficient with respect to a class of applications iff it is efficient for every application belonging to the class, it follows that a rule which is not efficient with respect to a class of applications can easily be efficient with respect to some subclass of applications which is found relevant in a particular context. Suppose that a particular context is such that only applications from some proper subset $\mathcal{B}$ of $\mathcal{A}$ can arise. In such a context, from the perspective of efficiency it is unnecessary to require that the rule yield efficient outcomes for every application belonging to $\mathcal{A}$; it would be adequate if the rule yields efficient outcomes for every application belonging to $\mathcal{B}$. Strict liability for instance is not an efficient rule with respect to $\mathcal{A}$ but is efficient with respect to the subclass of $\mathcal{A}$ which is characterized by the requirement that the optimal care for the victim be zero. Therefore it follows that in some particular contexts where the relevant sets of applications are proper subsets of $\mathcal{A}$ some h-liability rules with decoupled liability may turn out to be efficient.

It is known that there are no rules which are invariably efficient in the presence of errors or litigation or other costs not taken into account in the standard tort model. In a particular context some rules might perform better than others. The important point to note is that there is no presumption that the rules which are efficient within the framework of the standard tort model would necessarily perform better than those which are not. It is perfectly possible under a condition of this kind for a decoupled h-liability rule to perform better than any liability rule.

5 References

Barnes, David W. and Stout, Lynn A. (1992), The Economic Analysis of Tort Law, St. Paul, West Publishing Company.

Brown, John Prather (1973), 'Toward an Economic Theory of Liability', 2 Journal of Legal Studies, 323-350.

Calabresi, Guido (1961), 'Some Thoughts on Risk Distribution and the Law of Torts', 70 Yale Law Journal, 499-553.

Calabresi, Guido (1970), The Costs of Accidents: A Legal and Economic Analysis, New Haven, Yale University Press.

Coase, R.H. (1960), 'The Problem of Social Cost', 3 Journal of Law and Economics, 1-44.

Cooter, Robert D. (1985), 'Unity in Torts, Contracts and Property: The Model of Precaution', 73 California Law Review, 1-51.

Cooter, Robert D. (1991), 'Economic Theories of Legal Liability', 5 Journal of Economic Perspectives, 11-30.

Cooter, Robert D. and Ulen, Thomas S. (1999), Law and Economics, third edition, New York, Addison-Wesley.

Diamond, Peter A. (1974), 'Single Activity Accidents', 3 Journal of Legal Studies, 107-164.

Eisenberg, Melvin Aron (1988), The Nature of the Common Law, Cambridge, MA, Harvard University Press.

Jain, Satish K. and Singh, Ram (2002), 'Efficient Liability Rules: Complete Characterization', 75 *Journal of Economics* (*Zeitschrift für Nationalökonomie*), 105-124.

Landes, William M. and Posner, Richard A. (1987), The Economic Structure of Tort Law, Cambridge, MA, Harvard University Press.

Miceli, Thomas J. (1997), Economics of the Law: Torts, Contracts, Property, Litigation, Oxford, Oxford University Press.

Polinsky, A. Mitchell (2003), An Introduction to Law and Economics, third edition, Gaithersburg, MD, Aspen Publishers.

Posner, Richard A. (1972), 'A Theory of Negligence', 1 Journal of Legal Studies, 28-96.

Posner, Richard A. (1973), 'Strict Liability: A Comment', 2 Journal of Legal Studies, 205-221.

Posner, Richard A. (2007), Economic Analysis of Law, seventh edition, New York, NY, Wolters Kluwer Law & Business.

Shavell, Steven (1980), 'Strict Liability versus Negligence', 9 Journal of Legal Studies, 1-25.

Shavell, Steven (1987), Economic Analysis of Accident Law, Cambridge, MA, Harvard University Press.

6 Appendix

Lemma 1 *Let f be a h-liability rule; $f : [0,1]^2 \mapsto [0,\infty)^2$. If f is efficient for every application belonging to $\mathcal{A}$ then $[\forall p,q \in [0,1)][x(p,1) \geq 1 \wedge y(1,q) \geq 1]$.*

Proof: Let f be a h-liability rule; $f : [0,1]^2 \mapsto [0,\infty)^2$. Suppose: $[\exists q \in [0,1)][y(1,q) < 1]$. Let $y(1,q_0) < 1$.

Let $t > 0$. Choose r such that $0 \leq y(1,q_0)t < r < t$. Let $d_0 = \frac{r}{1-q_0}$. Let $c_0 > 0$ and $\epsilon > 0$.
Let C and D be specified as follows:
$C = \{0, c_0\}, D = \{0, q_0 d_0, d_0\}$.
For $(c,d) \in C \times D$, let $L(c,d)$ be as given in the following array[5]:

		d		
		0	$q_0 d_0$	d_0
c	0	$c_0 + \epsilon + t + q_0 d_0$	$c_0 + \epsilon + t$	$c_0 + \epsilon$
	c_0	$t + q_0 d_0$	t	0

$\epsilon > 0$ and $t > r = (1-q_0)d_0$ imply that $M = \{(c_0, d_0)\}$.
Let $(c^*, d^*) = (c_0, d_0)$.
Now,
$EC_2(c_0, d_0) = d_0$
$EC_2(c_0, q_0 d_0)$
$= q_0 d_0 + y(1,q_0)L(c_0, q_0 d_0)$
$= q_0 d_0 + y(1,q_0)t$
$EC_2(c_0, d_0) - EC_2(c_0, q_0 d_0)$
$= d_0 - q_0 d_0 - y(1,q_0)t$
$= (1-q_0)d_0 - y(1,q_0)t$
$= r - y(1,q_0)t$
> 0.
This implies that the unique total social cost minimizing configuration (c_0, d_0) is not a Nash equilibrium. f is therefore not efficient for the application under consideration.
If $[\exists p \in [0,1)][x(p,1) < 1]$ holds, then by an analogous argument one can demonstrate the existence of an application belonging to $\mathcal{A}$ for which f is not efficient.
The proposition is therefore established.

The following corollary follows immediately from Lemma 1.

[5] L has been specified in such a way that no inconsistency would arise even if $q_0 = 0$.

Corollary 1 *Let f be a h-liability rule; $f : [0,1]^2 \mapsto [0,\infty)^2$. If f is efficient for every application belonging to $\mathcal{A}$ then $\overline{s} \geq 1$.*

For every h-liability rule such that $[\exists p \in [0,1)][x(p,1) < 1] \vee [\exists q \in [0,1)][y(1,q) < 1]$, the proof of Lemma 1 shows the existence of an application belonging to $\mathcal{A}$ for which the configuration of costs of due care levels $(c^*, d^*) \in M$ is not a Nash equilibrium. Thus from the proof of Lemma 1 it follows that the following lemma holds.

Lemma 2 *Let f be a h-liability rule; $f : [0,1]^2 \mapsto [0,\infty)^2$. If f is such that for every application belonging to $\mathcal{A}$ the configuration of costs of due care levels $(c^*, d^*) \in M$ is a Nash equilibrium then $[\forall p, q \in [0,1)][x(p,1) \geq 1 \wedge y(1,q) \geq 1]$.*

The following corollary follows immediately from Lemma 2.

Corollary 2 *Let f be a h-liability rule; $f : [0,1]^2 \mapsto [0,\infty)^2$. If f is such that for every application belonging to $\mathcal{A}$ the configuration of costs of due care levels $(c^*, d^*) \in M$ is a Nash equilibrium then $\overline{s} \geq 1$.*

Lemma 3 *Let f be a h-liability rule; $f : [0,1]^2 \mapsto [0,\infty)^2$. If f is efficient for every application belonging to $\mathcal{A}$ then $[x^* \leq 1 \wedge y^* \leq 1]$.*

Proof: Suppose $x^* > 1$.
Choose $0 < \frac{1}{x^*} < t < 1$.
Choose $c_0, d_0, \epsilon, \delta_1, \delta_2 > 0$.

Let C and D be specified as follows:
$C = \{0, c_0, c_0 + \epsilon\}, D = \{0, d_0\}$.
For $(c,d) \in C \times D$, let $L(c,d)$ be as given in the following array:

		d: 0	d: d_0
c	0	$c_0 + \delta_1 + d_0 + \delta_2 + t\epsilon$	$c_0 + \delta_1 + t\epsilon$
	c_0	$d_0 + \delta_2 + t\epsilon$	$t\epsilon$
	$c_0 + \epsilon$	$d_0 + \delta_2$	0

$\delta_1 > 0$, $\delta_2 > 0$ and $0 < t < 1$ imply that (c_0, d_0) is the unique total social cost minimizing configuration, i.e., $M = \{(c_0, d_0)\}$.
Let $(c^*, d^*) = (c_0, d_0)$.
Now,
$EC_1(c_0, d_0) = c_0 + x^* t\epsilon$
$EC_1(c_0 + \epsilon, d_0) = c_0 + \epsilon$
$EC_1(c_0, d_0) - EC_1(c_0 + \epsilon, d_0) = c_0 + x^* t\epsilon - c_0 - \epsilon$
$= \epsilon[x^* t - 1]$
> 0

Thus the unique total social cost minimizing configuration of costs of care is not a Nash equilibrium; establishing that the rule is not efficient for the application in question.
If $y^* > 1$, an analogous argument shows that there exists an application belonging to $\mathcal{A}$ for which the rule is not efficient.
The lemma is therefore established.

The following corollary follows immediately from Lemma 3.

Corollary 3 *Let f be a h-liability rule; $f : [0,1]^2 \mapsto [0,\infty)^2$. If f is efficient for every application belonging to $\mathcal{A}$ then $\overline{s} \leq 2$.*

For every h-liability rule such that $[x^* > 1 \vee y^* > 1]$, the proof of Lemma 3 shows the existence of an application belonging to $\mathcal{A}$ for which the configuration of costs of due care levels $(c^*, d^*) \in M$ is not a Nash equilibrium. Thus from the proof of Lemma 3 it follows that the following lemma holds.

Lemma 4 *Let f be a h-liability rule; $f : [0,1]^2 \mapsto [0,\infty)^2$. If f is such that for every application belonging to $\mathcal{A}$ the configuration of costs of due care levels $(c^*, d^*) \in M$ is a Nash equilibrium then $[x^* \leq 1 \wedge y^* \leq 1]$.*

The following corollary follows immediately from Lemma 4.

Corollary 4 *Let f be a h-liability rule; $f : [0,1]^2 \mapsto [0,\infty)^2$. If f is such that for every application belonging to $\mathcal{A}$ the configuration of costs of due care levels $(c^*, d^*) \in M$ is a Nash equilibrium then $\overline{s} \leq 2$.*

Theorem 1 *Let f be a h-liability rule; $f : [0,1]^2 \mapsto [0,\infty)^2$. If f is efficient for every application belonging to $\mathcal{A}$ then $\overline{s} = 1$.*

Proof: Let h-liability rule; $f : [0,1]^2 \mapsto [0,\infty)^2$ be efficient for every application belonging to $\mathcal{A}$. Then from Corollaries 1 and 3 we have: $1 \leq \overline{s} \leq 2$.
Suppose $1 < \overline{s} \leq 2$.
We have: $x^* + y^* > 1$ as $\overline{s} > 1$.
Therefore we obtain: $[1 \geq x^* > 0 \wedge 1 \geq y^* > 0]$, in view of Lemma 3.
Let:
(i) $c_0, d_0, \epsilon_1, \epsilon_2 > 0$
(ii) $\beta > 0$
(iii) $\frac{x^*}{x^*+y^*}\beta < \alpha_1 < \theta_1 < x^*\beta$
(iv) $\frac{y^*}{x^*+y^*}\beta < \alpha_2 < \theta_2 < y^*\beta$.
Let C and D be specified as follows:
$C = \{0, c_0, c_0 + \theta_1\}, D = \{0, d_0, d_0 + \theta_2\}$.
For $(c,d) \in C \times D$, let $L(c,d)$ be as given in the following array:

		d		
		0	d_0	$d_0 + \theta_2$
	0	$c_0 + \theta_1 + d_0 + \theta_2 + \epsilon_1 + \epsilon_2$	$c_0 + \theta_1 + \theta_2 + \epsilon_1$	$c_0 + \theta_1 + \epsilon_1 + \alpha_2$
c	c_0	$\theta_1 + d_0 + \theta_2 + \epsilon_2$	$\theta_1 + \theta_2$	$\theta_1 + \alpha_2$
	$c_0 + \theta_1$	$d_0 + \theta_2 + \epsilon_2 + \alpha_1$	$\theta_2 + \alpha_1$	$\alpha_1 + \alpha_2 - \beta$

$\epsilon_1 > 0$, $\epsilon_2 > 0$, $\alpha_1 > 0$, $\alpha_2 > 0$ and $\alpha_1 + \alpha_2 - \beta > 0$ imply[6] that that $M = \{(c_0, d_0)\}$.
Let $(c^*, d^*) = (c_0, d_0)$.
Now,

$$EC_1(c_0 + \theta_1, d_0 + \theta_2) = c_0 + \theta_1 + x^*(\alpha_1 + \alpha_2 - \beta) \quad (1)$$
$$EC_1(c_0, d_0 + \theta_2) = c_0 + x^*(\theta_1 + \alpha_2) \quad (2)$$
$$EC_1(0, d_0 + \theta_2) = x(0,1)(c_0 + \theta_1 + \epsilon_1 + \alpha_2) \quad (3)$$
$$EC_2(c_0 + \theta_1, d_0 + \theta_2) = d_0 + \theta_2 + y^*(\alpha_1 + \alpha_2 - \beta) \quad (4)$$
$$EC_2(c_0 + \theta_1, d_0) = d_0 + y^*(\theta_2 + \alpha_1) \quad (5)$$
$$EC_2(c_0 + \theta_1, 0) = y(1,0)(d_0 + \theta_2 + \epsilon_2 + \alpha_1) \quad (6)$$
$$EC_1(c_0 + \theta_1, d_0 + \theta_2) - EC_1(c_0, d_0 + \theta_2) = [c_0 + \theta_1 + x^*(\alpha_1 + \alpha_2 - \beta)] - [c_0 + x^*(\theta_1 + \alpha_2)]$$
$$= (1 - x^*)\theta_1 + x^*\alpha_1 - x^*\beta$$
$$< (1 - x^*)\theta_1 + x^*\theta_1 - x^*\beta, \text{ as } \theta_1 > \alpha_1 > 0 \text{ and } x^* > 0$$
$$= \theta_1 - x^*\beta$$
$$< 0 \quad (7)$$

[6] Adding the inequalities $\frac{x^*}{x^*+y^*}\beta < \alpha_1$ and $\frac{y^*}{x^*+y^*}\beta < \alpha_2$, we obtain $\alpha_1 + \alpha_2 - \beta > 0$.

$$
\begin{aligned}
&EC_1(c_0+\theta_1, d_0+\theta_2) - EC_1(0, d_0+\theta_2)\\
&= [c_0+\theta_1+x^*(\alpha_1+\alpha_2-\beta)] - x(0,1)[c_0+\theta_1+\epsilon_1+\alpha_2]\\
&\leq [c_0+\theta_1+x^*(\alpha_1+\alpha_2-\beta)] - [c_0+\theta_1+\epsilon_1+\alpha_2], \text{ as } x(0,1)\geq 1 \text{ by Lemma 1}\\
&= x^*(\alpha_1-\beta) - (1-x^*)\alpha_2 - \epsilon_1\\
&< 0 \qquad (8)
\end{aligned}
$$

$$
\begin{aligned}
&EC_2(c_0+\theta_1, d_0+\theta_2) - EC_2(c_0+\theta_1, d_0) = [d_0+\theta_2+y^*(\alpha_1+\alpha_2-\beta)] - [d_0+y^*(\theta_2+\alpha_1)]\\
&= (1-y^*)\theta_2 + y^*\alpha_2 - y^*\beta\\
&< (1-y^*)\theta_2 + y^*\theta_2 - y^*\beta, \text{ as } \theta_2>\alpha_2>0 \text{ and } y^*>0\\
&= \theta_2 - y^*\beta\\
&< 0 \qquad (9)
\end{aligned}
$$

$$
\begin{aligned}
&EC_2(c_0+\theta_1, d_0+\theta_2) - EC_2(c_0+\theta_1, 0)\\
&= [d_0+\theta_2+y^*(\alpha_1+\alpha_2-\beta)] - y(1,0)[d_0+\theta_2+\epsilon_2+\alpha_1]\\
&\leq [d_0+\theta_2+y^*(\alpha_1+\alpha_2-\beta)] - [d_0+\theta_2+\epsilon_2+\alpha_1], \text{ as } y(1,0)\geq 1 \text{ by Lemma 1}\\
&= y^*(\alpha_2-\beta) - (1-y^*)\alpha_1 - \epsilon_2\\
&< 0 \qquad (10)
\end{aligned}
$$

(7)-(10) establish that $(c_0+\theta_1, d_0+\theta_2)$ is a Nash equilibrium. But $(c_0+\theta_1, d_0+\theta_2) \notin M$; which implies that f is not efficient for the application under consideration.

This contradiction establishes the theorem.

As a h-liability rule; $f : [0,1]^2 \mapsto [0,\infty)^2$ exhibits decoupled liability iff $\overline{s} \neq 1$, Theorem 1 can also be stated as follows:

Theorem 2 *If a h-liability rule; $f : [0,1]^2 \mapsto [0,\infty)^2$ exhibits decoupled liability then it is not the case that f is efficient for every application belonging to $\mathcal{A}$.*

Theorem 3 *Let f be a h-liability rule; $f : [0,1]^2 \mapsto [0,\infty)^2$. f has the property that for every application belonging to $\mathcal{A}$ the configuration of costs of due care levels $(c^*, d^*) \in M$ is a Nash equilibrium iff it satisfies the following two conditions:*
(i) $[\forall p, q \in [0,1)][x(p,1) \geq 1 \wedge y(1,q) \geq 1]$
(ii) $x^* \leq 1 \wedge y^* \leq 1$.

Proof: Suppose f has the property that for every application belonging to $\mathcal{A}$ the configuration of costs of due care levels $(c^*, d^*) \in M$ is a Nash equilibrium. Then (i) holds by

Lemma 2 and (ii) holds by Lemma 4.

Next, assume that f satisfies conditions (i) and (ii). Take any application $< C, D, \pi, H, (c^*, d^*) \in M >$ belonging to $\mathcal{A}$. Suppose (c^*, d^*) is not a Nash equilibrium. This implies:
$(\exists c' \in C)[c' + x[p(c'), q(d^*)]L(c', d^*) < c^* + x^* L^*] \vee (\exists d' \in D)[d' + y[p(c^*), q(d')]L(c^*, d') < d^* + y^* L^*]$. (1)
Suppose $(\exists c' \in C)[c' + x[p(c'), q(d^*)]L(c', d^*) < c^* + x^* L^*]$ holds. (2)
$c' < c^* \wedge (2) \rightarrow c' + L(c', d^*) < c^* + x^* L^*$, as $x[p(c'), q(d^*)] \geq 1$ by condition (i)
$\rightarrow c' + L(c', d^*) < c^* + L^*$, as $x^* \leq 1$ by condition (ii)
$\rightarrow c' + d^* + L(c', d^*) < c^* + d^* + L^*$
$\rightarrow TSC(c', d^*) < TSC(c^*, d^*)$.
This is a contradiction as total social costs are minimum at (c^*, d^*). Therefore we conclude:
$c' < c^* \rightarrow (2)$ cannot hold. (3)
For $c' > c^*$, we have: $x[p(c'), q(d^*)] = x(1, 1) = x^*$.
Consequently,
$c' > c^* \wedge (2) \rightarrow c' + x^* L(c', d^*) < c^* + x^* L^*$
$\rightarrow (1 - x^*)c' + x^*[c' + d^* + L(c', d^*)] < (1 - x^*)c^* + x^*[c^* + d^* + L^*]$
$\rightarrow (1 - x^*)c' < (1 - x^*)c^*$, as $TSC(c', d^*) \geq TSC(c^*, d^*)$. (4)
$(1 - x^*) > 0 \wedge (4) \rightarrow c' < c^*$, which contradicts the hypothesis that $c' > c^*$. (5)
$(1 - x^*) = 0 \wedge (4) \rightarrow 0 < 0$, a contradiction. (6)
(5) and (6) establish that (4) cannot hold. Therefore it follows that:
$c' > c^* \rightarrow (2)$ cannot hold. (7)
(3) and (7) establish that (2) cannot hold.
By an analogous argument one can show that $(\exists d' \in D)[d' + y[p(c^*), q(d')]L(c^*, d') < d^* + y^* L^*]$ cannot hold.
This establishes the theorem.