Structure of Incremental Liability Rules*

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Abstract

This paper investigates the structure of incremental liability rules. Necessary and sufficient conditions are derived for an incremental liability rule to be efficient. A liability rule, in the ordinary sense of the term, is a rule which specifies the proportions in which the loss, in case of accident, is to be apportioned between the victim and the injurer as a function of their proportions of nonnegligence. In contrast, an incremental liability rule is a rule which specifies (i) which of the two parties, the victim or the injurer, is to be the non-residual liability holder; and (ii) the proportion of the incremental loss, which can be ascribed to the negligence of the non-residual party, to be borne by the non-residual liability holder. The necessary and sufficient conditions for an incremental liability rule to be efficient, derived in the paper, can be stated as follows: Let the party which is the residual liability holder when both parties are nonnegligent be designated as r and the other party as nr. An incremental liability rule is efficient for every admissible application iff its structure is such that: (i) If party r is negligent and party nr is nonnegligent, then party r must remain the residual liability holder. (ii) If party nr is negligent and party r is nonnegligent, then party nr must either become the residual liability holder or liability of nr must be equal to the entire incremental loss which can be ascribed to the negligence of nr. The paper also discusses the significance of incremental liability rules from a normative perspective.

Keywords: Liability Rules, Incremental Liability Rules, Efficiency, Necessary and Sufficient Conditions for Efficiency, Incremental Loss, Residual Liability Holder.

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A central question in the economic analysis of tort law is how to apportion accident loss between victim and tortfeasor so that both parties involved in the harmful interaction are induced to take socially optimal levels of care. There is extensive literature on the efficiency of liability rules, the rules for apportioning losses between victims and injurers. Considerations relating to efficiency of liability rules have occupied an important place in the law and economics literature right from its inception. The pioneering contribution by Calabresi (1961) analyzed the effect of liability rules on parties' behaviour. In his seminal contribution Coase (1960) looked at liability rules from the point of view of their implications for social costs. The rule of negligence was analyzed by Posner (1972) from the perspective of economic efficiency. The first formal analysis of liability rules was done by Brown (1973). His main results demonstrated the efficiency of the rule of negligence and the rule of strict liability with the defense of contributory negligence; and the inefficiency of strict liability and no liability. Formal treatment of some of the most important results of the vast literature on liability rules is contained in Landes and Posner (1987) and Shavell (1987). A complete characterization of efficient liability rules is contained in Jain and Singh (2002).

In the literature dealing with the question of efficiency of liability rules, the problem has generally been considered within the framework of accidents resulting from interaction of two risk-neutral parties, the victim and the injurer. The social goal is taken to be the minimization of total social costs, which are defined to be the sum of costs of care taken by the two parties and expected accident loss. The probability of accident and the amount of loss in case of occurrence of accident are assumed to depend on the levels of care taken by the two parties. A liability rule determines the proportions in which the two parties are to bear the loss in case of occurrence of accident on the basis of whether and by what proportions the parties involved in the interaction were negligent. A liability rule is efficient iff it invariably induces both parties to behave in ways which result in socially optimal outcomes, i.e., outcomes under which total social costs are minimized. The central result regarding the efficiency question that has emerged is that a liability rule is efficient if and only if it satisfies the condition of negligence liability. The condition of negligence liability requires that in a two-party interaction if one party is nonnegligent and the other party is negligent then the entire loss, in case of occurrence of accident, must be borne by the negligent party.

Closely related to the idea of a liability rule is the idea of an incremental liability rule. An incremental liability rule is a rule which, as a function of proportions of nonnegligence, specifies (i) which of the two parties, the victim or the injurer, is to be the non-residual liability holder; and (ii) the proportion of the incremental loss, which can be ascribed to the negligence of the non-residual liability holder, to be borne by the non-residual party. The incremental liability rules, unlike the liability rules, have not received the requisite attention in the literature. It would be argued in the paper that incremental liability rules are of considerable significance from a normative perspective.

Although it is generally taken for granted in the literature that courts apportion accident loss between the parties by applying some liability rule or the other, this view is by no means entirely undisputed. For instance, Grady[1] (1983, 1984, 1989) has consistently, and very cogently, argued that when courts make use of the negligence rule for determining liability, they in fact apply the incremental variant of the negligence rule rather than the liability rule variant of it. If negligence rule is viewed as a liability rule then it would be defined by the requirements that in case of accident (i) the injurer bears no part of the loss iff she is nonnegligent; and (ii) the injurer bears the whole of the loss iff she is negligent. On the other hand, the incremental variant of the negligence rule is defined by the requirements that in case of accident (i) the injurer bears no part of the loss iff she is nonnegligent; and (ii) the injurer bears the incremental loss which can be ascribed to the negligence of the injurer iff she is negligent. A more concise but equivalent way to define the incremental version of the negligence rule is to define it by the requirement that in case of accident the injurer is to bear the incremental loss which can be ascribed to her negligence. If the courts in fact use incremental liability rules rather than liability rules then the importance of the incremental liability rules is immediate. In this paper we argue, as mentioned earlier, that the incremental liability rules are of considerable normative significance; and this significance is independent of whether the rules used by the courts are ordinary liability rules or their incremental versions.

[1] Grady's analysis of the negligence rule differs from the mainstream approach in two respects: (i) He considers the incremental version of the negligence rule rather than the usual liability version of it. (ii) The notion of negligence is usually defined as shortfall from the due care level. In Grady's formulation, however, the notion of negligence is defined in terms of cost-justified untaken precautions. It is important to note that the two ideas of incremental negligence rule and of negligence as untaken precaution, although related in Grady's analysis, are distinct and logically independent. The two can be used in conjunction as well as separately.

The main concern of this paper is with the question of efficiency of incremental liability rules. The necessary and sufficient conditions for an incremental liability rule to be efficient derived in this paper can be stated as follows: Let the party which is the residual liability holder when both parties are nonnegligent be designated as r and the other party as nr. An incremental liability rule is efficient for every admissible application iff its structure is such that: (i) If party r is negligent and party nr is nonnegligent, then party r must remain the residual liability holder. (ii) If party nr is negligent and party r is nonnegligent, then party nr must either become the residual liability holder or liability of nr must be equal to the entire incremental loss which can be ascribed to the negligence of nr.

The paper is divided into three sections. The first section contains the definitions and assumptions. The characterization of efficient incremental liability rules is discussed in section 2. The last section concludes with some remarks on the normative significance of the incremental liability rules. The formal statements and proofs of propositions are relegated to the Appendix.

1 Definitions and Assumptions

We consider accidents resulting from interaction of two parties, assumed to be strangers to each other, in which, to begin with, the entire loss falls on one party to be called the victim (plaintiff). The other party would be referred to as the injurer (defendant). At times, the victim would be referred to as individual 1; and the injurer as individual 2. We denote by $\alpha \geq 0$ the index of the level of care taken by the victim; and by $\beta \geq 0$ the index of the level of care taken by the injurer.

Let

$A = \{\alpha \mid \alpha \geq 0$ is the index of some feasible level of care which can be taken by the victim$\}$, and

$B = \{\beta \mid \beta \geq 0$ is the index of some feasible level of care which can be taken by the injurer$\}$.

We assume:

$$0 \in A \ \wedge \ 0 \in B. \tag{A1}$$

We denote by $c(\alpha)$ the cost to the victim of care level α and by $d(\beta)$ the cost to the injurer of care level β.

Let $C = \{c(\alpha) \mid \alpha \in A\}$, and $D = \{d(\beta) \mid \beta \in B\}$.

We assume:

$$c(0) = 0 \ \wedge \ d(0) = 0. \tag{A2}$$

We also assume that c and d are strictly increasing functions of α and β respectively.

(A3)

In view of (A2) and (A3) it follows that:
$(\forall c \in C)(c \geq 0) \ \wedge \ (\forall d \in D)(d \geq 0)$.
A consequence of (A3) is that c and d themselves can be taken as indices of levels of care of the two parties.

Let π denote the probability of occurrence of accident and $H \geq 0$ the loss in case of occurrence of accident. Both π and H will be assumed to be functions of c and d; $\pi = \pi(c, d), H = H(c, d)$. Let $L = \pi H$. L is thus the expected loss due to accident.
We assume:

$$(\forall c, c' \in C)(\forall d, d' \in D)[[c > c' \ \rightarrow \ \pi(c, d) \leq \pi(c', d)] \ \wedge \ [d > d' \ \rightarrow \ \pi(c, d) \leq \pi(c, d')]]. \tag{A4}$$

and

$$(\forall c, c' \in C)(\forall d, d' \in D)[[c > c' \ \rightarrow \ H(c, d) \leq H(c', d)] \ \wedge \ [d > d' \ \rightarrow \ H(c, d) \leq H(c, d')]]. \tag{A5}$$

In other words, it is assumed that a larger expenditure on care by either party, given the expenditure on care by the other party, does not result in greater probability of occurrence of accident or in greater accident loss.

From A(4) and A(5) it follows that:
$(\forall c, c' \in C)(\forall d, d' \in D)[[c > c' \ \rightarrow \ L(c, d) \leq L(c', d)] \ \wedge \ [d > d' \ \rightarrow \ L(c, d) \leq L(c, d')]]$.

That is to say: a larger expenditure on care by either party, given the expenditure on care by the other party, results in lesser or equal expected accident loss.

Total social costs (TSC) are defined to be the sum of cost of care by the victim, cost of care by the injurer, and the expected loss due to accident; $TSC = c + d + L(c, d)$. Let $M = \{(c', d') \in C \times D \mid c' + d' + L(c', d')$ is minimum of $\{c + d + L(c, d) \mid c \in C \ \wedge \ d \in D\}\}$. Thus M is the set of all costs of care configurations (c', d') which are total social cost minimizing. It will be assumed that:
C, D and L are such that M is nonempty. (A6)

The notion of negligence is defined in terms of shortfall from a reference point, called the due care level. A party's level of care is characterized as negligent or otherwise depending on whether it is less than the due care level or not. Let c^* and d^*, where $(c^*, d^*) \in M$, denote the due care levels of the victim and the injurer respectively. We define nonnegligence functions p and q as follows:

$p : C \mapsto [0, 1]$ such that[2] :

$$\begin{aligned} p(c) &= \tfrac{c}{c^*} \quad \text{if } c < c^*; \\ &= 1 \quad \text{if } c \geq c^* \end{aligned}$$

$q : D \mapsto [0, 1]$ such that:

$$\begin{aligned} q(d) &= \tfrac{d}{d^*} \quad \text{if } d < d^*; \\ &= 1 \quad \text{if } d \geq d^*. \end{aligned}$$

In case there is a legally binding due care level for the plaintiff, it would be taken to be identical with c^* figuring in the definition of function p; and in case there is a legally binding due care level for the defendant, it would be taken to be identical with d^* figuring in the definition of function q. Thus implicitly it is being assumed that the legally binding due care levels are always set appropriately from the point of view of minimizing total social costs.

p and q would be interpreted as proportions of nonnegligence of the victim and the injurer respectively. The victim would be called negligent if $p < 1$ and nonnegligent if $p = 1$. Similarly, the injurer would be called negligent if $q < 1$ and nonnegligent if $q = 1$.

A liability rule, in the ordinary sense of the term, is a rule which specifies the proportions in which the loss, in case of accident, is to be apportioned between the two parties as a function of their proportions of nonnegligence. In other words, a liability rule is a function g from I^2 to I^2, $g : I^2 \mapsto I^2$, such that: $g(p, q) = (u, v)$, where $u =$ proportion of loss to be borne by the victim, $v =$ proportion of loss to be borne by the injurer, and $u + v = 1$.

In the sequel we define the notion of an incremental liability rule.

Corresponding to each $(c, d) \in C \times D$, we define:

$$\begin{aligned} \hat{L}_1(c, d) &= L(c, d) - L(c^*, d) \quad \text{if} \quad L(c, d) - L(c^*, d) > 0 \\ &= 0 \quad \text{if} \quad L(c, d) - L(c^*, d) \leq 0 \end{aligned}$$

and

$$\begin{aligned} \hat{L}_2(c, d) &= L(c, d) - L(c, d^*) \quad \text{if} \quad L(c, d) - L(c, d^*) > 0 \\ &= 0 \quad \text{if} \quad L(c, d) - L(c, d^*) \leq 0 \end{aligned}$$

[2]We use the standard notation to denote:
$\{x \mid 0 \leq x \leq 1\}$ by $[0, 1]$, $\{x \mid 0 \leq x < 1\}$ by $[0, 1)$, $\{x \mid 0 < x \leq 1\}$ by $(0, 1]$ and $\{x \mid 0 < x < 1\}$ by $(0, 1)$.

An incremental liability rule is a rule which specifies (i) which of the two parties, the victim (1) or the injurer (2), is to be the non-residual liability holder; and (ii) the proportion of the incremental loss, which can be ascribed to the negligence of the non-residual party, to be borne by the non-residual liability holder. Formally, an incremental liability rule is a function f from I^2 to $\{1,2\} \times I$, $f : I^2 \mapsto \{1,2\} \times I$, such that: $f(p,q) = (a, b_a)$.
We will write x for b_1; and y for b_2.

Let C, D, π and H be given. If accident takes place and loss of $H(c,d)$ materializes, then:

If $a[p(c), q(d)] = 1$ then the loss borne by the victim will be:
$= x[p(c), q(d)][H(c,d) - H(c^*, d)\frac{\pi(c^*,d)}{\pi(c,d)}]$ if $H(c,d) - H(c^*, d)\frac{\pi(c^*,d)}{\pi(c,d)} > 0$ and $\pi(c,d) \neq 0$
$= 0$ otherwise;
and the remainder of the loss will be borne by the injurer.

If $a[p(c), q(d)] = 2$ then the loss borne by the injurer will be:
$= y[p(c), q(d)][H(c,d) - H(c, d^*)\frac{\pi(c,d^*)}{\pi(c,d)}]$ if $H(c,d) - H(c, d^*)\frac{\pi(c,d^*)}{\pi(c,d)} > 0$ and $\pi(c,d) \neq 0$
$= 0$ otherwise;
and the remainder of the loss will be borne by the victim.

The expected costs of the victim and the injurer, to be denoted by EC_1 and EC_2 respectively, therefore, are:
$[c + x[p(c), q(d)]\hat{L}_1(c,d)]$ and $[d + L(c,d) - x[p(c), q(d)]\hat{L}_1(c,d)]$ respectively, if $a[p(c), q(d)] = 1$; and
$[c + L(c,d) - y[p(c), q(d)]\hat{L}_2(c,d)]$ and $[d + y[p(c), q(d)]\hat{L}_2(c,d)]$ respectively, if $a[p(c), q(d)] = 2$.
Both parties are assumed to prefer smaller expected costs to larger expected costs and be indifferent between alternatives with equal expected costs.

Let f be an incremental liability rule. An application of f consists of specification of C, D, π, H and $(c^*, d^*) \in M$ satisfying (A1)-(A6). The class of all applications satisfying (A1)-(A6) will be denoted by $\mathcal{A}$.

f is defined to be efficient for a given application $< C, D, \pi, H, (c^*, d^*) \in M >$ iff $(\forall(\overline{c}, \overline{d}) \in C \times D)[(\overline{c}, \overline{d})$ is a Nash equilibrium $\rightarrow (\overline{c}, \overline{d}) \in M]$ and $(\exists(\overline{c}, \overline{d}) \in C \times D)[(\overline{c}, \overline{d})$ is a Nash equilibrium]. In other words, an incremental liability rule is efficient for a given application iff (i) every $(\overline{c}, \overline{d}) \in C \times D$ which is a Nash equilibrium is total social cost minimizing, and (ii) there exists at least one $(\overline{c}, \overline{d}) \in C \times D$ which is a Nash equilibrium. f is defined to be efficient with respect to a class of applications iff it is efficient for every application belonging to that class.

The following example illustrates some of the above ideas.

Example 1

Let incremental liability rule f be the negligence rule defined by:[3] $(\forall p \in [0,1])(\forall q \in [0,1))[f(p,q) = (2,1)] \wedge (\forall p \in [0,1])[f(p,1) = (2,0)]$.
Consider an application of f such that:
$C = D = \{0,1,2\}$
For $(c,d) \in C \times D$, let $L(c,d)$ be as given in the following array:

		d		
		0	1	2
	0	10.0	7.0	5.5
c	1	7.0	4.0	2.5
	2	5.5	2.5	1.0

Here $(2,2)$ is the unique TSC-minimizing configuration of costs of care.
Let $(c^*, d^*) = (2,2)$.

The following array gives, for $(c,d) \in C \times D$, $EC(c,d) = (EC_1(c,d), EC_2(c,d))$:

		d		
		0	1	2
	0	$(5.5, 4.5)$	$(5.5, 2.5)$	$(5.5, \underline{2})$
c	1	$(3.5, 4.5)$	$(3.5, 2.5)$	$(3.5, \underline{2})$
	2	$(\underline{3}, 4.5)$	$(\underline{3}, 2.5)$	$(\underline{3}, \underline{2})$

Therefore $(2,2)$ is the only $(c,d) \in C \times D$ which is a Nash equilibrium. Thus f is efficient for this application.

[3] If the nonnegligence proportion of the non-residual party (a) is 1 then $\hat{L}_a = 0$; and consequently the entire expected loss would be borne by the residual party regardless of what b_a is. Therefore the negligence rule can be written in different but equivalent ways. For instance, the definition of rule of negligence given above is equivalent to the definition:
$(\forall p \in [0,1])(\forall q \in [0,1])[f(p,q) = (2,1)]$.

2 Characterization of Efficient Incremental Liability Rules

Incremental liability rules which are efficient for all applications belonging to $\mathcal{A}$ are characterized by the following conditions:
If $a(1,1) = 1$ then :
(i) $[a(1,q) = 1]$ for every $q < 1$ and
(ii) $[a(p,1) = 1 \rightarrow x(p,1) = 1]$ for every $p < 1$

If $a(1,1) = 2$ then :
(i) $[a(p,1) = 2]$ for every $p < 1$ and
(ii) $[a(1,q) = 2 \rightarrow y(1,q) = 1]$ for every $q < 1$.

The formal statement of the characterization theorem and its proof are contained in the Appendix.

The above necessary and sufficient conditions for efficiency can be restated as follows: Let the party which is the residual liability holder when both parties are nonnegligent be designated by r and the other party by nr. An incremental liability rule is efficient for every application belonging to $\mathcal{A}$ iff its structure is such that: (i) If party r is negligent and party nr is nonnegligent, then party r must remain the residual liability holder. (ii) If party nr is negligent and party r is nonnegligent, then party nr must either become the residual liability holder or liability of nr must be equal to the entire incremental loss which can be ascribed to its negligence.

In view of the fact that when both parties are nonnegligent, the entire loss falls on the residual liability holder, the necessary and sufficient conditions for efficiency imply that an incremental liability rule is efficient for all applications belonging to $\mathcal{A}$ iff: (i) If party r is negligent and party nr is nonnegligent, then the entire loss must be borne by party r. (ii) If party nr is negligent and party r is nonnegligent, then party nr must either bear the entire loss or must bear the entire incremental loss which can be ascribed to its negligence.

It is interesting to compare the necessary and sufficient conditions for efficiency of incremental liability rules with the necessary and sufficient conditions for efficiency of ordinary liability rules. An ordinary liability rule is efficient for all applications belonging to $\mathcal{A}$ iff it satisfies the condition of negligence liability. The condition of negligence liability requires that: (i) whenever the injurer is nonnegligent and the victim is negligent, the entire loss in case of accident must be borne by the victim, and (ii) whenever the victim is nonnegligent and the injurer is negligent, the entire loss in case of accident must be borne by the injurer. More formally, the condition of negligence liability is defined as follows:

Condition of Negligence Liability (NL): A liability rule g satisfies the condition of negligence liability iff $[[\forall p \in [0,1)][g(p,1) = (1,0)] \;\wedge\; [\forall q \in [0,1)][g(1,q) = (0,1)]]$.

While the efficiency-characterizing conditions in the two cases are similar, two important differences should be noted. Negligence liability condition which is both necessary and sufficient for a liability rule to be efficient does not impose any restrictions on the liability assignments when both parties are nonnegligent or both parties are negligent. For incremental liability rules also the necessary and sufficient conditions for efficiency do not impose any restrictions on liability assignments when both parties are negligent. However, when both parties are nonnegligent, from the very definition of an incremental liability rule it follows that the entire loss must be borne by one of the two parties alone. Secondly, when one party is negligent and the other nonnegligent, in the case of liability rules the requirement imposed by efficiency is symmetric with respect to the two parties. But in the case of incremental lability rules the treatment of the two parties is asymmetric. But once again the asymmetry is due to the very notion of an incremental liability rule.

A liability rule can be symmetric with respect to the two parties; but it is not possible for an incremental liability rule to be symmetric with respect to the two parties. The notion of symmetry with respect to the parties for liability rules and incremental liability rules can formally be defined as follows:

A liability rule g satisfies symmetry (S) iff $(\forall p, p', q, q', u, u', v, v' \in [0,1])[g(p,q) = (u,v) \wedge g(p',q') = (u',v') \wedge p' = q \wedge q' = p \rightarrow u' = v \wedge v' = u]$.

An incremental liability rule f satisfies symmetry (S-I) iff $(\forall p, p', q, q', z \in [0,1])[p' = q \wedge q' = p \rightarrow [a(p,q) = 1 \wedge b_1(p,q) = z \rightarrow a(p',q') = 2 \wedge b_2(p',q') = z] \wedge [a(p,q) = 2 \wedge b_2(p,q) = z \rightarrow a(p',q') = 1 \wedge b_1(p',q') = z]]$.

There are liability rules which satisfy symmetry; and also which violate symmetry. For instance the liability rule g defined by:
$(\forall p, q \in [0,1))[g(p,q) = (\frac{1}{2}, \frac{1}{2})] \wedge (\forall p \in [0,1))[g(p,1) = (1,0)] \wedge (\forall q \in [0,1))[g(1,q) = (0,1)] \wedge [g(1,1) = (\frac{1}{2}, \frac{1}{2})]$
satisfies symmetry; and the rule g given by:
$(\forall p \in [0,1])(\forall q \in [0,1))[g(p,q) = (0,1)] \wedge (\forall p \in [0,1])[g(p,1) = (1,0)]$
violates symmetry.

In contrast, no incremental liability rule can satisfy symmetry. This is immediate in view of the facts that while incremental liability rule being a function requires that $a(1,1)$ must be unique, symmetry (S-I) requires that $a(1,1)$ cannot be 1 or 2 without being the other as well.

3 Concluding Remarks

This paper has shown that what is required for efficiency of incremental liability rules, although not much dissimilar from what is required for efficiency of ordinary liability rules, is asymmetric with respect to the two parties. This asymmetry is potentially of considerable significance. The efficiency requirement for ordinary liability rules is symmetric with respect to the two parties. One implication of this is that if one wants to view the harmful interaction as essentially asymmetric in the sense of being caused by the injurer, then the requirement for efficiency can easily conflict with the intuitive requirement for justice. The notion of incremental liability rule is potentially of great use in such contexts as by making the victim as the non-residual liability holder, one can have efficiency without any significant sacrifice of requirement for justice.

The asymmetry of incremental liability rules is of significance not only for reducing conflicts between efficiency and intuitive notions of justice but also for enhancing efficiency in certain contexts. For instance, if the context is such that in the event of a large loss one of the parties is likely to be judgment-proof then, by the choice of an appropriate incremental liability rule in which the party likely to be judgment-proof is the non-residual liability holder, the possibilities of socially inappropriate behaviour on the part of the party likely to be judgment-proof could be attenuated or even eliminated altogether. Although both the incremental and the ordinary variants of the negligence rule are efficient under the standard assumptions and neither is efficient in the presence of errors in the determination of actual care levels, Grady has argued that in the presence of uncertainty in the determination of negligence the inducement for taking socially wasteful excessive care is less in the case of the incremental negligence rule than in the case of the ordinary negligence rule. Thus, potentially both the asymmetric and the incremental aspects of incremental liability rules can be made use of for enhancing efficiency in contexts where all of the assumptions of the standard tort model do not hold.

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5 Appendix

Proposition 1 *Let incremental liability rule f be efficient with respect to $\mathcal{A}$. Then we must have:*
$[a(1,1) = 1 \rightarrow (\forall q \in [0,1))[a(1,q) = 1]]$.

Proof: Let incremental liability rule f be efficient with respect to $\mathcal{A}$. Suppose $[a(1,1) = 1 \wedge (\exists q_0 \in [0,1))[a(1,q_0) = 2]]$.
Let (i) $0 < \epsilon_1, \epsilon_2, c_0, d_0$ and (ii) $\epsilon_2 < L_0$.
Let C and D be specified as follows:
$C = \{0, c_0\}, D = \{0, q_0 d_0, d_0\}$.
For $(c,d) \in C \times D$, let $L(c,d)$ be as given in the following array:

		d	
	0	q_0d_0	d_0
c = 0	$c_0+\epsilon_1+d_0+\epsilon_2+L_0$	$c_0+\epsilon_1+(1-q_0)d_0+\epsilon_2+L_0$	$c_0+\epsilon_1+L_0$
c = c_0	$d_0+\epsilon_2+L_0$	$(1-q_0)d_0+\epsilon_2+L_0$	L_0

It should be noted that the above specification of C, D and L is consistent with (A1)-(A6). Furthermore, the specification of L is done in such a way that no inconsistency would arise even if $q_0 = 0$.

As $\epsilon_1 > 0$ and $\epsilon_2 > 0$, it follows that (c_0, d_0) is the unique total social cost minimizing configuration. Let $(c^*, d^*) = (c_0, d_0)$.
Now, expected costs of the injurer at $(c_0, q_0d_0) = EC_2(c_0, q_0d_0)$
$= q_0d_0 + y(1, q_0)\hat{L}_2(c_0, q_0d_0)$, as $a(1, q_0) = 2$
$= q_0d_0 + y(1, q_0)[L(c_0, q_0d_0) - L(c_0, d_0)]$
$= q_0d_0 + y(1, q_0)[L_0 + (1 - q_0)d_0 + \epsilon_2 - L_0]$
$\leq q_0d_0 + (1 - q_0)d_0 + \epsilon_2$
$= d_0 + \epsilon_2$

$EC_2(c_0, d_0)$
$= d_0 + L(c_0, d_0) - x(1, 1)\hat{L}_1(c_0, d_0)$, as $a(1, 1) = 1$
$= d_0 + L_0$

Therefore,
$EC_2(c_0, d_0) - EC_2(c_0, q_0d_0)$
$\geq L_0 - \epsilon_2$
> 0.

This establishes that (c_0, d_0) is not a Nash equilibrium. Consequently f is not an efficient incremental liability rule for every application belonging to $\mathcal{A}$; establishing the proposition.

Analogously the following proposition can be established:

Proposition 2 *Let incremental liability rule f be efficient with respect to $\mathcal{A}$. Then we must have:*
$[a(1, 1) = 2 \rightarrow (\forall p \in [0, 1))[a(p, 1) = 2]]$.

Proposition 3 *Let incremental liability rule f be efficient with respect to $\mathcal{A}$. Then we must have:*
$[a(1,1) = 1 \rightarrow (\forall p \in [0,1))[a(p,1) = 1 \rightarrow x(p,1) = 1]]$.

Proof: Let incremental liability rule f be efficient with respect to $\mathcal{A}$. Suppose $[a(1,1) = 1 \wedge (\exists p_0 \in [0,1))[a(p_0,1) = 1 \wedge x(p_0,1) < 1]]$.

Choose positive numbers $c_0, d_0, L_0, \epsilon_1, \epsilon_2$ such that:
if $x(p_0,1) > 0$ then $\epsilon_1 < (1-p_0)c_0\frac{1-x(p_0,1)}{x(p_0,1)}$.

Now consider the following application belonging to $\mathcal{A}$:
$C = \{0, p_0c_0, c_0\}, D = \{0, d_0\}$.
For $(c,d) \in C \times D$, let $L(c,d)$ be as given in the following array:

		d	
		0	d_0
	0	$c_0 + d_0 + \epsilon_1 + \epsilon_2 + L_0$	$c_0 + \epsilon_1 + L_0$
c	p_0c_0	$(1-p_0)c_0 + d_0 + \epsilon_1 + \epsilon_2 + L_0$	$(1-p_0)c_0 + \epsilon_1 + L_0$
	c_0	$d_0 + \epsilon_2 + L_0$	L_0

In view of the fact that $\epsilon_1 > 0$ and $\epsilon_2 > 0$, it follows that (c_0, d_0) is the unique total social cost minimizing configuration. Let $(c^*, d^*) = (c_0, d_0)$.

Expected costs of the victim at $(c_0, d_0) = EC_1(c_0, d_0) = c_0$, as $a(1,1) = 1$

$EC_1(p_0c_0, d_0)$
$= p_0c_0 + x(p_0,1)\hat{L}_1(p_0c_0, d_0)$, as $a(p_0,1) = 1$
$= p_0c_0 + x(p_0,1)[L(p_0c_0, d_0) - L(c_0, d_0)]$
$= p_0c_0 + x(p_0,1)[(1-p_0)c_0 + \epsilon_1]$

$EC_1(c_0, d_0) - EC_1(p_0c_0, d_0)$
$= c_0 - p_0c_0 - x(p_0,1)[(1-p_0)c_0 + \epsilon_1]$
$= (1-p_0)c_0 - x(p_0,1)[(1-p_0)c_0 + \epsilon_1]$
$= (1 - x(p_0,1))(1-p_0)c_0 - x(p_0,1)\epsilon_1$

If $x(p_0,1) > 0$ then $(1-x(p_0,1))(1-p_0)c_0 - x(p_0,1)\epsilon_1 = x(p_0,1)[\frac{(1-x(p_0,1))}{x(p_0,1)}(1-p_0)c_0 - \epsilon_1] > 0$
If $x(p_0,1) = 0$ then $(1-x(p_0,1))(1-p_0)c_0 - x(p_0,1)\epsilon_1 = (1-p_0)c_0 > 0$

Thus $EC_1(c_0, d_0) - EC_1(p_0c_0, d_0) > 0$; and consequently it follows that (c_0, d_0) is not a Nash equilibrium. Therefore f is not an efficient incremental liability rule for every application belonging to $\mathcal{A}$; establishing the proposition.

Analogously the following proposition can be established:

Proposition 4 *Let incremental liability rule f be efficient with respect to $\mathcal{A}$. Then we must have:*
$[a(1,1) = 2 \rightarrow (\forall q \in [0,1))[a(1,q) = 2 \rightarrow y(1,q) = 1]]$.

Proposition 5 *Let f be an incremental liability rule satisfying the following four conditions:*
(i) $a(1,1) = 1 \rightarrow (\forall q \in [0,1))[a(1,q) = 1]$
(ii) $a(1,1) = 2 \rightarrow (\forall p \in [0,1))[a(p,1) = 2]$
(iii) $a(1,1) = 1 \rightarrow (\forall p \in [0,1))[a(p,1) = 1 \rightarrow x(p,1) = 1]$
(iv) $a(1,1) = 2 \rightarrow (\forall q \in [0,1))[a(1,q) = 2 \rightarrow y(1,q) = 1]$.
Let $< C, D, \pi, H, (c^*, d^*) \in M >$ *be an application belonging to $\mathcal{A}$. Then (c^*, d^*) is a Nash equilibrium.*

Proof: Let f be an incremental liability rule satisfying conditions (i)-(iv); and let $< C, D, \pi, H, (c^*, d^*) \in M >$ be an application belonging to $\mathcal{A}$.

First suppose $a(1,1) = 1$.

$EC_1(c^*, d^*) = c^*$
$c < c^* \wedge a(\frac{c}{c^*}, 1) = 1 \rightarrow EC_1(c, d^*) = c + [L(c, d^*) - L(c^*, d^*)]$, by (iii)
Therefore, $c < c^* \wedge a(\frac{c}{c^*}, 1) = 1 \rightarrow EC_1(c, d^*) - EC_1(c^*, d^*) = [c + L(c, d^*)] - [c^* + L(c^*, d^*)] = [c + d^* + L(c, d^*)] - [c^* + d^* + L(c^*, d^*)] = TSC(c, d^*) - TSC(c^*, d^*) \geq 0$ (1)
$c < c^* \wedge a(\frac{c}{c^*}, 1) = 2 \rightarrow EC_1(c, d^*) = c + L(c, d^*)$
Therefore, $c < c^* \wedge a(\frac{c}{c^*}, 1) = 2 \rightarrow EC_1(c, d^*) - EC_1(c^*, d^*) = [c + L(c, d^*)] - [c^*] = [c + d^* + L(c, d^*)] - [c^* + d^*] \geq [c + d^* + L(c, d^*)] - [c^* + d^* + L(c^*, d^*)] = TSC(c, d^*) - TSC(c^*, d^*) \geq 0$ (2)
$c > c^* \rightarrow EC_1(c, d^*) = c$
Therefore, $c > c^* \rightarrow EC_1(c, d^*) - EC_1(c^*, d^*) = c - c^* > 0$ (3)

$EC_2(c^*, d^*) = d^* + L(c^*, d^*)$
$d < d^* \rightarrow EC_2(c^*, d) = d + L(c^*, d)$, as $a(1, \frac{d}{d^*}) = 1$ by (i)
Therefore, $d < d^* \rightarrow EC_2(c^*, d) - EC_2(c^*, d^*) = [d + L(c^*, d)] - [d^* + L(c^*, d^*)] = [c^* + d + L(c^*, d)] - [c^* + d^* + L(c^*, d^*)] = TSC(c^*, d) - TSC(c^*, d^*) \geq 0$ (4)
$d > d^* \rightarrow EC_2(c^*, d) = d + L(c^*, d)$
Therefore, $d > d^* \rightarrow EC_2(c^*, d) - EC_2(c^*, d^*) = [d + L(c^*, d)] - [d^* + L(c^*, d^*)] = [c^* + d + L(c^*, d)] - [c^* + d^* + L(c^*, d^*)] = TSC(c^*, d) - TSC(c^*, d^*) \geq 0$ (5)

(1)-(5) establish that if $a(1, 1) = 1$ then (c^*, d^*) is a Nash equilibrium. (6)
By an analogous argument it can be shown that if $a(1, 1) = 2$ then (c^*, d^*) is a Nash equilibrium. (7)
(6) and (7) establish the proposition.

Proposition 6 *Let f be an incremental liability rule satisfying the following four conditions:*
(i) $a(1, 1) = 1 \rightarrow (\forall q \in [0, 1))[a(1, q) = 1]$
(ii) $a(1, 1) = 2 \rightarrow (\forall p \in [0, 1))[a(p, 1) = 2]$
(iii) $a(1, 1) = 1 \rightarrow (\forall p \in [0, 1))[a(p, 1) = 1 \rightarrow x(p, 1) = 1]$
(iv) $a(1, 1) = 2 \rightarrow (\forall q \in [0, 1))[a(1, q) = 2 \rightarrow y(1, q) = 1]$.
Let $< C, D, \pi, H, (c^*, d^*) \in M >$ *be an application belonging to* $\mathcal{A}$. *Then:*
$(\forall(\overline{c}, \overline{d}) \in C \times D)[(\overline{c}, \overline{d})$ *is a Nash equilibrium* $\rightarrow (\overline{c}, \overline{d}) \in M]$.

Proof: Let f be an incremental liability rule satisfying (i)-(iv); and let $< C, D, \pi, H, (c^*, d^*) \in M >$ be an application belonging to $\mathcal{A}$. Suppose $(\overline{c}, \overline{d})$ is a Nash equilibrium.

$(\overline{c}, \overline{d})$ is a Nash equilibrium implies
$EC_1(\overline{c}, \overline{d}) \leq EC_1(c^*, \overline{d})$ (1)
and
$EC_2(\overline{c}, \overline{d}) \leq EC_2(\overline{c}, d^*)$ (2)
(1) $\wedge$ (2) $\rightarrow \overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \leq EC_1(c^*, \overline{d}) + EC_2(\overline{c}, d^*)$ (3)

First suppose $a(1, 1) = 1$.

Now,

$$
\begin{aligned}
EC_1(c^*, \overline{d}) &= c^* \quad \text{if } d < d^*; \text{as } a(1, \tfrac{d}{d^*}) = 1 \\
&= c^* \quad \text{if } d \geq d^*; \text{as } a(1, 1) = 1
\end{aligned}
$$

Thus, $EC_1(c^*, \overline{d}) = c^*$ (4)

$$\begin{aligned} EC_2(\overline{c}, d^*) &= d^* + L(\overline{c}, d^*) \leq d^* + L(c^*, d^*) && \text{if } \overline{c} \geq c^*; \text{as } a(1,1) = 1 \\ &= d^* + L(\overline{c}, d^*) - \hat{L}_1(\overline{c}, d^*) = d^* + L(c^*, d^*) && \text{if } \overline{c} < c^* \text{and } a(p,1) = 1 \\ &= d^* && \text{if } \overline{c} < c^* \text{and } a(p,1) = 2 \end{aligned}$$

Thus, $EC_2(\overline{c}, d^*) \leq d^* + L(c^*, d^*)$ (5)

(3)-(5) imply that:

$$TSC(\overline{c}, \overline{d}) = \overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \leq EC_1(c^*, \overline{d}) + EC_2(\overline{c}, d^*) \leq c^* + d^* + L(c^*, d^*) = TSC(c^*, d^*) \quad (6)$$

As total social costs are minimized at (c^*, d^*), it follows that $TSC(\overline{c}, \overline{d}) = TSC(c^*, d^*)$; and consequently we must have $(\overline{c}, \overline{d}) \in M$. An analogous argument shows that if $a(1,1) = 2$ then also we must have $(\overline{c}, \overline{d}) \in M$. The proposition therefore stands established.

Theorem 1 *An incremental liability rule is efficient for every application belonging to $\mathcal{A}$ iff it satisfies the following four conditions:*

(i) $a(1,1) = 1 \rightarrow (\forall q \in [0,1))[a(1,q) = 1]$
(ii) $a(1,1) = 2 \rightarrow (\forall p \in [0,1))[a(p,1) = 2]$
(iii) $a(1,1) = 1 \rightarrow (\forall p \in [0,1))[a(p,1) = 1 \rightarrow x(p,1) = 1]$
(iv) $a(1,1) = 2 \rightarrow (\forall q \in [0,1))[a(1,q) = 2 \rightarrow y(1,q) = 1]$.

Proof Let f be an incremental liability rule; and suppose that f satisfies (i)-(iv). Let $< C, D, \pi, H, (c^*, d^*) \in M >$ be an arbitrary application belonging to $\mathcal{A}$. Then, (c^*, d^*) is a Nash equilibrium by Proposition 5 and we have $(\forall(\overline{c}, \overline{d}) \in C \times D)[(\overline{c}, \overline{d})$ is a Nash equilibrium $\rightarrow (\overline{c}, \overline{d}) \in M]$ by Proposition 6. Thus, f is efficient for every application belonging to $\mathcal{A}$. This establishes sufficiency of (i)-(iv) for an incremental liability rule to be efficient with respect to $\mathcal{A}$. Propositions 1-4 establish the necessity of (i)-(iv) respectively for an incremental liability rule to be efficient for every application belonging to $\mathcal{A}$.