Efficiency of Liability Rules with Multiple Victims*

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Abstract

This paper investigates the structure of liability rules from the efficiency perspective when there are multiple victims. It is shown that, when there is one injurer and multiple victims, there is no liability rule with the property of invariably yielding efficient outcomes. The fact that there is no rule which is efficient for all applications of course does not in any way preclude the possibility of a rule being efficient with respect to some subclass of applications which may be of interest. We consider in the paper the important subclass of applications ($\mathcal{A}'$) which are such that expected loss of a victim depends only on the care level taken by that victim and the care level taken by the injurer. It is shown that a sufficient condition for a one-injurer multiple-victim liability rule to be efficient with respect to the above subclass of applications $\mathcal{A}'$ is that its structure be such that: (i) whenever the injurer is negligent and a particular victim is nonnegligent, the entire loss incurred by that victim must be borne by the injurer; and (ii) whenever a particular victim is negligent and the injurer is nonnegligent, the entire loss incurred by that victim must be borne by the victim himself. In fact, for an important subclass of one-injurer multiple-victim liability rules, characterized by the condition that the proportions in which the loss incurred by a particular victim is to be borne by the injurer and that victim must depend only on the nonnegligence proportions of the injurer and that victim, the above condition is both necessary and sufficient for efficiency with respect to the restricted subclass of applications $\mathcal{A}'$.

Keywords: Liability Rules, Multiple-Victim Liability Rules, Efficient Liability Rules, Negligence Liability, Strict Liability, Negligence Rule

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In the economic analysis of law legal rules are analyzed from the perspective of efficiency. That is to say, the question that is asked about a legal rule is whether its structure is such that when rational individuals act within its framework, they are induced to act in ways so that the social outcome which comes about as a consequence of the totality of actions undertaken by the individuals is invariably efficient. In the last few decades a large part of contemporary law has been analyzed to determine whether the relevant legal rules and procedures have the characteristic of always giving rise to efficient outcomes. In the tort law, which deals with harmful interactions, an important question is how to apportion accident loss among the parties involved in the harmful interaction so that all the parties are induced to take socially optimal levels of care for accident prevention and loss reduction in case of occurrence of accident. The rules which are used to apportion accident loss among parties involved in harmful interaction, called liability rules, have been extensively analyzed from the perspective of efficiency. The framework which has generally been adopted for dealing with the question of efficiency of liability rules is that of accidents resulting from interaction of risk-neutral parties. Minimization of total social costs is taken to be the social goal. Total social costs are defined as the sum of costs of care taken by the parties and expected accident loss. The probability of accident and the amount of loss in case of occurrence of accident are assumed to depend on the levels of care taken by the parties. A liability rule determines the proportions in which the parties are to bear the loss in case of occurrence of accident on the basis of whether and by what proportions the parties involved in the interaction were negligent. A liability rule is efficient for a particular application iff it induces the parties to behave in ways which result in a socially optimal outcome, i.e., an outcome under which total social costs are minimized; and a liability rule is efficient with respect to a set of applications iff it is efficient for every application belonging to the set.

Most of the contributions relating to the question of efficiency of liability rules have been obtained in the context of two-party interactions involving one victim and one tortfeasor. The pioneering contributions in the area were made by Calabresi (1961), Coase (1960) and Posner (1972). The first formal analysis of liability rules was done by Brown (1973). He showed that the rule of negligence and the rule of strict liability with the defense of contributory negligence have the property of inducing both the victim and the injurer to take socially optimal levels of care. Detailed analysis of the important liability rules is contained in Landes and Posner (1987) and Shavell (1987). A complete characterization of efficient liability rules has been obtained in Jain and Singh (2002). For liability rules defined for one injurer and one victim, the main result which has emerged is that a liability rule is efficient for all applications iff it satisfies the condition of negligence liability. The condition of negligence liability requires that in a two-party interaction if one party is nonnegligent and the other negligent then the entire loss, in case of occurrence of accident, must be borne by the negligent party.

In the context of multi-party interactions, the first results were obtained by Landes and Posner (1980). They showed that the negligence rule defined for one victim and multiple injurers is efficient for all applications. One-victim multiple-tortfeasor context was also analyzed in Tietenberg (1989), Kornhauser and Revsez (1989) and Miceli and Segerson (1991). In Jain and Kundu (2006) a sufficient condition has been derived for efficiency of any one-victim multiple-tortfeasor liability rule. It is shown there that if a one-victim multiple-tortfeasor liability rule satisfies the condition of collective negligence liability then it must be efficient for all applications. The condition of collective negligence liability requires that whenever some individuals are negligent, no nonnegligent individual bears any loss in case of occurrence of accident. This condition, while sufficient for any one-victim multiple-tortfeasor liability rule to be efficient, is also necessary for efficiency of any simple liability rule defined for one victim and multiple injurers. Under a simple liability rule the liability shares depend only on the negligence or otherwise of parties and not on the extent of negligence. Most of the liability rules used in practice are simple liability rules. An important exception is the comparative negligence rule.

Unlike the case of one victim and multiple injures, the case of one injurer and multiple victims, arguably at least as important as the former, if not more, has not received the requisite attention. The purpose of this paper is to investigate the structure of one-tortfeasor multi-victim liability rules from the efficiency perspective. It turns out that, when there are multiple victims and one tortfeasor, there is no liability rule which is efficient for all applications. The fact that there is no rule which is efficient for all applications does not of course in any way preclude the possibility of a rule being efficient with respect to some subclass of applications which may be of interest. We consider in the paper the important subclass of applications ($\mathcal{A}'$) which are such that the expected loss of a particular victim depends only on the care level taken by that victim and the care level taken by the injurer. It is shown that a sufficient condition for a one-injurer multiple-victim liability rule to be efficient with respect to subclass $\mathcal{A}'$ of applications is that its structure be such that: (i) whenever the injurer is negligent and a particular victim is nonnegligent the entire loss incurred by that victim must be borne by the injurer; and (ii) whenever a particular victim is negligent and the injurer is nonnegligent the entire loss incurred by that victim must be borne by the victim himself. In fact, for an important subclass of one-injurer multiple-victim liability rules, characterized by the condition that the proportions in which the loss incurred by a particular victim is to be borne by the injurer and that victim must depend only on the nonnegligence proportions of the injurer and that victim, the above condition is both necessary and sufficient for efficiency with respect to subclass of applications $\mathcal{A}'$.

The paper is divided into four sections. Section 1 sets out the framework within which the efficiency problem is analyzed. Section 2 states and proves the impossibility theorem. The next section contains the efficiency analysis of one-injurer multi-victim liability rules when applications are restricted to set $\mathcal{A}'$. The concluding section discusses the differences between the efficiency conditions for one-victim multi-injurer liability rules and one-injurer multi-victim liability rules and the reasons thereof.

1 Definitions and Assumptions

We consider accidents involving one injurer (individual 0) and n victims (individuals $1, \ldots, n$); where $n \geq 2$. Let $N = \{0, 1, \ldots, n\}$ and $N_V = \{1, \ldots, n\}$. It would be assumed that the losses, to begin with, fall on the victims. We denote by $a_i, i \in N$, the index of the level of care taken by individual i. For each $i \in N$, let $A_i = \{a_i \mid a_i$ is the index of some feasible level of care which can be taken by individual $i\}$. We assume:

Assumption A1 $(\forall i \in N)[(\forall a_i \in A_i)(a_i \geq 0) \ \wedge \ 0 \in A_i]$.

For each $i \in N$, we denote by $c_i(a_i)$ the cost of individual i's care level a_i. Let
$C_i = \{c_i(a_i) \mid a_i \in A_i\},\ i \in N$.
We assume:

Assumption A2 $(\forall i \in N)[c_i(0) = 0]$.

Furthermore, it would be assumed that:

Assumption A3 $(\forall i \in N)[(\forall a_i, a_i' \in A_i)[a_i > a_i' \;\; \rightarrow \;\; c_i(a_i) > c_i(a_i')]]$.

In other words, c_i is assumed to be a strictly increasing function of $a_i, i \in N$.

Assumptions (A2) and (A3) imply that: $(\forall i \in N)(\forall c_i \in C_i)(c_i \geq 0)$.

In view of Assumption (A3), for each $i \in N, c_i$ itself can be taken to be an index of level of care taken by individual i. Let π denote the probability of occurrence of accident and $H_i \geq 0$ the loss to individual $i \in N_V$ in case of occurrence of accident. π and $H_i, i \in N_V$, will be assumed to be functions of $c_0, c_1, \ldots, c_n$; $\pi = \pi(c_0, c_1, \ldots, c_n)$; $H_i = H_i(c_0, c_1, \ldots, c_n)$. Let $\mathcal{L}_i = \pi H_i, i \in N_V$. $\mathcal{L}_i, i \in N_V$, is thus a function of $c_0, c_1, \ldots, c_n$; and denotes the expected loss to individual i. We assume:

Assumption A4 $(\forall (c_0, c_1, \ldots, c_n), (c_0', c_1', \ldots, c_n') \in C_0 \times C_1 \times \ldots \times C_n)(\forall j \in N)[(\forall i \in N)(i \neq j \;\; \rightarrow \;\; c_i = c_i') \wedge c_j > c_j' \;\; \rightarrow \;\; \pi(c_0, c_1, \ldots, c_n) \leq \pi(c_0', c_1', \ldots, c_n')]$.

Assumption A5 $(\forall k \in N_V)(\forall (c_0, c_1, \ldots, c_n), (c_0', c_1', \ldots, c_n') \in C_0 \times C_1 \times \ldots \times C_n)(\forall j \in N)[(\forall i \in N)(i \neq j \;\; \rightarrow \;\; c_i = c_i') \wedge c_j > c_j' \;\; \rightarrow \;\; H_k(c_0, c_1, \ldots, c_n) \leq H_k(c_0', c_1', \ldots, c_n')]$.

That is to say, greater care by an individual, given the care levels of all other individuals, does not result in greater probability of accident or greater loss to some victim in case of accident.

Assumptions (A4) and (A5) imply:
$(\forall k \in N_V)(\forall (c_0, c_1, \ldots, c_n), (c_0', c_1', \ldots, c_n') \in C_0 \times C_1 \times \ldots \times C_n)(\forall j \in N)[(\forall i \in N)(i \neq j \;\; \rightarrow \;\; c_i = c_i') \wedge c_j > c_j' \;\; \rightarrow \;\; \mathcal{L}_k(c_0, c_1, \ldots, c_n) \leq \mathcal{L}_k(c_0', c_1', \ldots, c_n')]$.
That is to say, greater care by an individual, given the levels of care of all other individuals, results, for every $k \in N_V$, in lesser or equal expected accident loss.

Total social costs (TSC) are defined to be the sum of costs of care of all the individuals and expected losses of the victims; TSC $= \Sigma_{i \in N} c_i + \Sigma_{i \in N_V} \mathcal{L}_i(c_0, c_1, \ldots, c_n)$. Total social costs are thus a function of $c_0, c_1, \ldots, c_n$. Let $M = \{(c_0', c_1', \ldots, c_n') \mid \Sigma_{i \in N} c_i' + \Sigma_{i \in N_V} \mathcal{L}_i(c_0', c_1', \ldots, c_n')$ is minimum of $\{\Sigma_{i \in N} c_i + \Sigma_{i \in N_V} \mathcal{L}_i(c_0, c_1, \ldots, c_n) \mid (c_0, c_1, \ldots, c_n) \in C_0 \times C_1 \times \ldots \times C_n\}\}$. Thus M is the set of all costs of care configurations $(c_0', c_1', \ldots, c_n')$ which are total social cost minimizing. It will be assumed that:

Assumption A6 *$C_0, C_1, \ldots, C_n; \mathcal{L}_1, \mathcal{L}_2, \ldots, \mathcal{L}_n$ are such that M is nonempty.*

Let $(c_0^*, c_1^*, \ldots, c_n^*) \in M$. Given $c_0^*, c_1^*, \ldots, c_n^*$, we define for each $i \in N$, function p_i, $p_i : C_i \mapsto [0,1]$, as follows:[1]
$p_i(c_i) = 1$ if $c_i \geq c_i^*$
$p_i(c_i) = \frac{c_i}{c_i^*}$ if $c_i < c_i^*$.[2]
Depending on the liability rule, there could be legally specified due care levels for all individuals, or for some of them or for none of them. If there is a legally specified due care level for individual $i, i \in N$, then c_i^* used in the definition of p_i would be taken to be identical with the legally specified due care level. If there is no legally specified due care level for individual i then c_i^* used in the definition of p_i can be taken to be any $c_i^* \in C_i$ subject to the requirement that $(c_0^*, c_1^*, \ldots, c_n^*) \in M$. Thus in all cases, for each individual i, c_i^* would denote the legally binding due care level for individual i whenever the idea of legally binding due care level for individual i is applicable.[3]
$p_i(c_i) = 1$ would be interpreted as meaning that individual i is taking at least the due care and $p_i(c_i) < 1$ as meaning that individual i is taking less than the due care. If $p_i(c_i) = 1$, individual i would be called nonnegligent; and if $p_i(c_i) < 1$, individual i would be called negligent.

A one-tortfeasor multi-victim liability rule (to be written as (1,n)-liability rule in abbreviated form) is a rule which specifies the proportions in which each victim's loss is to be divided between the injurer and the victim in question in case of occurrence of accident as a function of proportions of nonnegligence of individuals. Formally, a (1,n)-liability rule is a function f from $[0,1]^{n+1}$ to $[0,1]^n$, $f : [0,1]^{n+1} \mapsto [0,1]^n$, such that: $f(p_0, p_1, \ldots, p_n) = f[p_0(c_0), p_1(c_1), \ldots, p_n(c_n)] = (x_1, \ldots, x_n) = [x_1(p_0(c_0), p_1(c_1), \ldots, p_n(c_n)), \ldots,$
$x_n(p_0(c_0), p_1(c_1), \ldots, p_n(c_n))]$, where $x_i, i \in N_V$, is the proportion of loss to the victim i which is borne by victim i; and $(1 - x_i) = y_i$, the proportion to be borne by the injurer.

If accident takes place and losses of $H_1, \ldots, H_n$ are incurred by victims $1, \ldots, n$ respectively then $x_1[p_0(c_0), p_1(c_1), \ldots, p_n(c_n)]H_1(c_0, c_1, \ldots, c_n), \ldots, x_n[p_0(c_0), p_1(c_1), \ldots, p_n(c_n)]$
$H_n(c_0, c_1, \ldots, c_n)$ will be borne by individuals $1, \ldots, n$ respectively; and

[1] We use the standard notation to denote $\{x \mid 0 \leq x \leq 1\}$ by $[0,1]$.

[2] If $c_i < c_i^*$ then we must have: $c_i^* > 0$; as $(\forall i \in N)(\forall c_i \in C_i)(c_i \geq 0)$.

[3] Thus, implicitly it is being assumed that the legally specified due care levels are in all cases consistent with the objective of total social cost minimization. This standard assumption is crucial for results on the efficiency of liability rules.

$\Sigma_{i\in N_V} y_i[p_0(c_0), p_1(c_1), \ldots, p_n(c_n)]H_i(c_0, c_1, \ldots, c_n)$ will be borne by the injurer. As, to begin with, in case of occurrence of accident, the losses fall on the victims $y_i[p_0(c_0), p_1(c_1), \ldots, p_n(c_n)]H_i(c_0, c_1, \ldots, c_n)$, represents the liability payment by the injurer to victim $i, i \in N_V$.

Victim i's, $i \in N_V$, expected costs therefore are:

$c_i + x_i[p_0(c_0), p_1(c_1), \ldots, p_n(c_n)]\mathcal{L}_i(c_0, c_1, \ldots, c_n)$;

and injurer's expected costs are:

$c_0 + \Sigma_{i\in N_V} y_i[p_0(c_0), p_1(c_1), \ldots, p_n(c_n)]\mathcal{L}_i(c_0, c_1, \ldots, c_n)$.

Every individual belonging to N is assumed to regard an outcome to be at least as good as another outcome iff expected costs of the individual under the former are less than or equal to expected costs under the latter.

The context in which a (1,n)-liability rule f is applied is completely specified by $C_0, C_1, \ldots, C_n$; $\mathcal{L}_1, \ldots, \mathcal{L}_n$; and $(c_0^*, c_1^*, \ldots, c_n^*) \in M$. The set of all applications $< C_0, C_1, \ldots, C_n; \mathcal{L}_1, \ldots, \mathcal{L}_n; (c_0^*, c_1^*, \ldots, c_n^*) \in M >$ satisfying Assumptions $(A1) - (A6)$ will be denoted by $\mathcal{A}$.

Now we introduce a condition on expected loss functions:

Victims' expected loss functions satisfy the condition of mutual independence (CMI) iff $(\forall k \in N_V)(\forall (c_0, c_1, \ldots, c_n), (c_0', c_1', \ldots, c_n') \in C_0 \times C_1 \times \ldots \times C_n)[c_0 = c_0' \wedge c_k = c_k' \rightarrow \mathcal{L}_k(c_0, c_1, \ldots, c_n) = \mathcal{L}_k(c_0', c_1', \ldots, c_n')]$.

In other words, victims' expected loss functions satisfy the condition of mutual independence iff every victim's expected loss depends only on his own care level and the care level of the injurer.[4]

The set of all applications $< C_0, C_1, \ldots, C_n; \mathcal{L}_1, \ldots, \mathcal{L}_n; (c_0^*, c_1^*, \ldots, c_n^*) \in M >$ satisfying Assumptions $(A1) - (A6)$ and CMI will be denoted by $\mathcal{A}'$.

If victims' expected loss functions satisfy CMI then we will write $\mathcal{L}_i(c_0, c_1, \ldots, c_n)$ as $L_i(c_0, c_i)$, $i \in N_V$.

Let $\mathcal{F}$ designate the set of all (1,n)-liability rules. Let the subclass $\mathcal{F}'$ of $\mathcal{F}$ be defined by the condition: $(\forall k \in N_V)(\forall (p_0, p_1, \ldots, p_n), (p_0', p_1', \ldots, p_n') \in [0, 1]^{n+1})[p_0 = p_0' \wedge p_k = p_k' \rightarrow x_k(p_0, p_1, \ldots, p_n) = x_k(p_0', p_1', \ldots, p_n')]$. Thus, if a (1,n)-liability rule belongs to $\mathcal{F}'$ then the proportions in which victim k's loss is to be divided between the injurer and victim k in case of accident is entirely determined by the nonnegligence proportions of the injurer

[4]If condition CMI holds then it must be the case that the probability of accident π depends only on the care level of the injurer.

and victim k.

A (1,n)-liability rule f is defined to be efficient for a given application $< C_0, C_1, \ldots, C_n; \mathcal{L}_1, \ldots, \mathcal{L}_n; (c_0^*, c_1^*, \ldots, c_n^*) \in M >$ iff $(\forall(\overline{c}_0, \overline{c}_1, \ldots, \overline{c}_n) \in C_0 \times C_1 \times \ldots \times C_n)[(\overline{c}_0, \overline{c}_1, \ldots, \overline{c}_n)$ is a Nash equilibrium $\rightarrow$ $(\overline{c}_0, \overline{c}_1, \ldots, \overline{c}_n) \in M]$ $\wedge$ $(\exists(\overline{c}_0, \overline{c}_1, \ldots, \overline{c}_n) \in C_0 \times C_1 \times \ldots \times C_n)[(\overline{c}_0, \overline{c}_1, \ldots, \overline{c}_n)$ is a Nash equilibrium]. In other words, a (1,n)-liability rule is efficient for a given application iff (i) every configuration $(\overline{c}_0, \overline{c}_1, \ldots, \overline{c}_n) \in C_0 \times C_1 \times \ldots \times C_n$ which is a Nash equilibrium is total social cost minimizing and (ii) there exists at least one configuration $(\overline{c}_0, \overline{c}_1, \ldots, \overline{c}_n) \in C_0 \times C_1 \times \ldots \times C_n$ which is a Nash equilibrium. A (1,n)-liability rule is efficient with respect to a set of applications iff it is efficient for every application belonging to that set.

The following examples illustrate some of the concepts discussed above:

Example 1 Let (1,2)-liability rule f be defined by:
$(\forall(p_0, p_1, p_2) \in [0,1]^3)[[p_0 < 1 \rightarrow x_1(p_0, p_1, p_2) = 0 \wedge x_2(p_0, p_1, p_2) = 0] \wedge [p_0 = 1 \rightarrow x_1(p_0, p_1, p_2) = 1 \wedge x_2(p_0, p_1, p_2) = 1]]$.[5]

Consider the following application of the above rule:
$C_0 = C_1 = C_2 = \{0, 1\}$;
$\mathcal{L}_1(0,0,0) = \mathcal{L}_1(0,0,1) = 2; \mathcal{L}_1(0,1,0) = \mathcal{L}_1(0,1,1) = .75; \mathcal{L}_1(1,0,0) = \mathcal{L}_1(1,0,1) = 1.25; \mathcal{L}_1(1,1,0) = \mathcal{L}_1(1,1,1) = 0$;
$\mathcal{L}_2(0,0,0) = \mathcal{L}_2(0,1,0) = 2; \mathcal{L}_2(0,0,1) = \mathcal{L}_2(0,1,1) = .75; \mathcal{L}_2(1,0,0) = \mathcal{L}_2(1,1,0) = 1.25; \mathcal{L}_2(1,0,1) = \mathcal{L}_2(1,1,1) = 0$.

$(1,1,1)$ is the unique TSC-minimizing configuration of costs of care. Let $(c_0^*, c_1^*, c_2^*) = (1,1,1)$.
Here $(1,1,1)$ is the only $(c_0, c_1, c_2) \in C_0 \times C_1 \times C_2$, which is a Nash equilibrium. The rule is therefore efficient for the application under consideration.

Example 2 Consider the following application of the rule of Example 1:
$C_0 = C_1 = C_2 = \{0, 1\}$;
$(\forall(c_0, c_1, c_2) \in C_0 \times C_1 \times C_2)[\mathcal{L}_1(c_0, c_1, c_2) = \mathcal{L}_2(c_0, c_1, c_2) = 2.25 - .75\Sigma_{i \in N} c_i]$.

[5] This rule can be thought of as the negligence rule defined for one injurer and two victims.

$(1,1,1)$ is the unique TSC-minimizing configuration of costs of care. Let $(c_0^*, c_1^*, c_2^*) = (1,1,1)$.
Here $(1,0,0)$ is the only $(c_0, c_1, c_2) \in C_0 \times C_1 \times C_2$, which is a Nash equilibrium. The rule is therefore inefficient for the application under consideration.

2 Impossibility Theorem

First we establish that there is no liability rule defined for one injurer and multiple victims which invariably gives rise to efficient outcomes.

Theorem 1 *There is no (1,n)-liability rule which is efficient for all applications belonging to $\mathcal{A}$.*

Proof: Let f be any (1,n)-liability rule.
Consider the application belonging to $\mathcal{A}$ specified by:
$(\forall i \in N)[C_i = \{0,1\}]$.
Let:
$\frac{1}{2} < \beta < 1$;
$0 < \epsilon < \frac{1-\beta}{n}$.
Let:
$(\forall i \in N_V - \{1,2\})(\forall (c_0, c_1, \ldots, c_n) \in C_0 \times C_1 \times \ldots \times C_n)[\mathcal{L}_i(c_0, c_1, \ldots, c_n) = \beta + \epsilon - \epsilon c_0 - \beta c_i]$;
$(\forall i \in \{1,2\})(\forall (c_0, c_1, \ldots, c_n) \in C_0 \times C_1 \times \ldots \times C_n)[\mathcal{L}_i(c_0, c_1, \ldots, c_n) = 2\beta + \epsilon - \epsilon c_0 - \beta c_1 - \beta c_2]$.

Therefore, we obtain:
$TSC(c_0, c_1, \ldots, c_n) = \Sigma_{i \in N} c_i + \Sigma_{i \in \{1,2\}}[2\beta + \epsilon - \epsilon c_0 - \beta c_1 - \beta c_2] + \Sigma_{i \in N_V - \{1,2\}}[\beta + \epsilon - \epsilon c_0 - \beta c_i]$
$= (n+2)\beta + n\epsilon + (1 - n\epsilon)c_0 + (1 - 2\beta)c_1 + (1 - 2\beta)c_2 + \Sigma_{i \in N_V - \{1,2\}}(1 - \beta)c_i$.
$(1 - n\epsilon) > 0$, $(1 - 2\beta) < 0$ and $(1 - \beta) > 0$ imply that TSC is uniquely minimized at $[c_0 = 0, c_1 = 1, c_2 = 1, (\forall i \in N_V - \{1,2\})(c_i = 0)]$.
Let $[c_0^* = 0 \wedge c_1^* = 1 \wedge c_2^* = 1 \wedge (\forall i \in N_V - \{1,2\})(c_i^* = 0)]$.

Consider the configuration $(c_0^*, c_1^*, \ldots, c_n^*)$.
Given that every $i \in N, i \neq 1$, is going to use $c_i = c_i^*$;

If victim 1 uses $c_1 = 0$, his expected costs $= EC_1(c_0^*, 0, c_2^*, \ldots, c_n^*) =$
$0 + x_1[p_0(c_0^*), p_1(0), p_2(c_2^*), \ldots, p_n(c_n^*)]\mathcal{L}_1[c_0^*, 0, c_2^*, \ldots, c_n^*]$
$= x_1[p_0(c_0^*), p_1(0), p_2(c_2^*), \ldots, p_n(c_n^*)](\beta + \epsilon)$
$\leq \beta + \epsilon$, as $0 \leq x_1[p_0(c_0^*), p_1(0), p_2(c_2^*), \ldots, p_n(c_n^*)] \leq 1$
< 1, as $\epsilon < 1 - \beta$

If victim 1 uses $c_1 = 1 = c_1^*$, his expected costs $= EC_1(c_0^*, c_1^*, c_2^*, \ldots, c_n^*) =$
$1 + x_1[p_0(c_0^*), p_1(c_1^*), p_2(c_2^*), \ldots, p_n(c_n^*)]\mathcal{L}_1[c_0^*, c_1^*, c_2^*, \ldots, c_n^*]$
$= 1 + x_1[p_0(c_0^*), p_1(c_1^*), p_2(c_2^*), \ldots, p_n(c_n^*)]\epsilon$
≥ 1.

Thus, given that every $i \in N, i \neq 1$, is going to use $c_i = c_i^*$; for victim 1 $c_1 = 0$ is better than $c_1 = 1 = c_1^*$. Therefore it follows that the unique total social cost minimizing configuration of care levels $(c_0^*, c_1^*, \ldots, c_n^*)$ is not a Nash equilibrium. f is therefore inefficient with respect to $\mathcal{A}$.

3 Efficiency of Rules with Restricted Domain of Applicability

Now we introduce a condition on (1,n)-liability rules.

Condition of (1,n)-Negligence Liability [(1,n)-NL]: A (1,n)-liability rule f satisfies the condition of (1,n)-negligence liability iff $(\forall k \in N_V)(\forall (p_0, p_1, \ldots, p_n) \in [0,1]^{n+1})[[p_0 < 1 \wedge p_k = 1 \rightarrow x_k(p_0, p_1, \ldots, p_n) = 0] \wedge [p_0 = 1 \wedge p_k < 1 \rightarrow x_k(p_0, p_1, \ldots, p_n) = 1]]$.

In other words, a (1,n)-liability rule satisfies the condition of (1,n)-negligence liability iff its structure is such that for every $k \in N_V$: (i) whenever the injurer is negligent and victim k is nonnegligent then the entire loss incurred by victim k must be borne by the injurer; and (ii) whenever the injurer is nonnegligent and victim k is negligent then the entire loss incurred by victim k must be borne by victim k himself.

In the sequel we show sufficiency of (1,n)-negligence liability for efficiency of any (1,n)-liability rule with respect to set of applications $\mathcal{A}'$.

Lemma 1 *Let (1,n)-liability rule f satisfy the condition of (1,n)-negligence liability. Let $< C_0, C_1, \ldots, C_n; \mathcal{L}_1, \ldots, \mathcal{L}_n; (c_0^*, c_1^*, \ldots, c_n^*) \in M >$ be an application belonging to $\mathcal{A}'$. Then, $(c_0^*, c_1^*, \ldots, c_n^*)$ is a Nash equilibrium.*

Proof: Let (1,n)-liability rule f satisfy the condition of (1,n)-negligence liability. Consider any application $< C_0, C_1, \ldots, C_n; \mathcal{L}_1, \ldots, \mathcal{L}_n; (c_0^*, c_1^*, \ldots, c_n^*) \in M >$ belonging to $\mathcal{A}'$. Suppose $(c_0^*, c_1^*, \ldots, c_n^*)$ is not a Nash equilibrium. Then, for some $k \in N$ there is some $c_k' \in C_k$ which is a better strategy for individual k than c_k^*, given that every other individual i uses $c_i^*, i \in N, i \neq k$. That is to say, if $k = 0$, we must have:

$$(\exists c_0' \in C_0)[c_0' + \Sigma_{i \in N_V} y_i[p_0(c_0'), p_1(c_1^*), \ldots, p_n(c_n^*)] L_i(c_0', c_i^*) \; < \; c_0^* + \Sigma_{i \in N_V} y_i[p_0(c_0^*), p_1(c_1^*), \ldots, p_n(c_n^*)] L_i(c_0^*, c_i^*)] \quad (1)$$

and if $k \in N_V$, we must have:

$$(\exists c_k' \in C_k)[c_k' + x_k[p_0(c_0^*), p_1(c_1^*), \ldots, p_k(c_k'), \ldots, p_n(c_n^*)] L_k(c_0^*, c_k') \; < \; c_k^* + x_k[p_0(c_0^*), p_1(c_1^*), \ldots, p_k(c_k^*), \ldots, p_n(c_n^*)] L_k(c_0^*, c_k^*)]. \quad (2)$$

Suppose (1) holds and $c_0' < c_0^*$.
$c_0' < c_0^* \rightarrow (\forall i \in N_V)\ [y_i[p_0(c_0'), p_1(c_1^*), \ldots, p_n(c_n^*)] = 1]$, by condition (1,n)-NL. Therefore:
$(1) \wedge c_0' < c_0^* \; \rightarrow \; c_0' + \Sigma_{i \in N_V} L_i(c_0', c_i^*) \; < \; c_0^* + \Sigma_{i \in N_V} y_i[p_0(c_0^*), p_1(c_1^*), \ldots, p_n(c_n^*)] L_i(c_0^*, c_i^*)$
$\rightarrow \; c_0' + \Sigma_{i \in N_V} L_i(c_0', c_i^*) \; < \; c_0^* + \Sigma_{i \in N_V} L_i(c_0^*, c_i^*)$, as $0 \leq y_i[p_0(c_0^*), p_1(c_1^*), \ldots, p_n(c_n^*)] \leq 1, i \in N_V$.
Adding $\Sigma_{i \in N_V} c_i^*$ to both sides we obtain:
$TSC(c_0', c_1^*, \ldots, c_n^*) < TSC(c_0^*, c_1^*, \ldots, c_n^*)$,
a contradiction as TSC is minimum at $(c_0^*, c_1^*, \ldots, c_n^*)$. (3)

Next suppose (1) holds and $c_0' > c_0^*$.
First we note that $[p_0(c_0'), p_1(c_1^*), \ldots, p_n(c_n^*)] = [p_0(c_0^*), p_1(c_1^*), \ldots, p_n(c_n^*)] = (1, \ldots, 1)$. For $i \in N_V$, designate $x_i(1, \ldots, 1)$ by x_i^*, and $y_i(1, \ldots, 1)$ by y_i^*.
(1) and $c_0' > c_0^* \rightarrow c_0' + \Sigma_{i \in N_V} y_i^* L_i(c_0', c_i^*) \; < \; c_0^* + \Sigma_{i \in N_V} y_i^* L_i(c_0^*, c_i^*)$
$\rightarrow c_0' \; < \; c_0^* + \Sigma_{i \in N_V} y_i^* [L_i(c_0^*, c_i^*) - L_i(c_0', c_i^*)]$.
Now, $(\forall i \in N_V)[L_i(c_0^*, c_i^*) - L_i(c_0', c_i^*) \geq 0]$, as $c_0' > c_i^*$.
As $(\forall i \in N_V)[0 \leq y_i^* \leq 1]$:
$c_0' \; < \; c_0^* + \Sigma_{i \in N_V} y_i^* [L_i(c_0^*, c_i^*) - L_i(c_0', c_i^*)] \rightarrow c_0' \; < \; c_0^* + \Sigma_{i \in N_V} [L_i(c_0^*, c_i^*) - L_i(c_0', c_i^*)]$
$\rightarrow c_0' + \Sigma_{i \in N_V} L_i(c_0', c_i^*) \; < \; c_0^* + \Sigma_{i \in N_V} L_i(c_0^*, c_i^*)$.
Adding $\Sigma_{i \in N_V} c_i^*$ to both sides we obtain:
$TSC(c_0', c_1^*, \ldots, c_n^*) < TSC(c_0^*, c_1^*, \ldots, c_n^*)$, a contradiction. (4)

Next suppose (2) holds and $c'_k < c^*_k, k \in N_V$.
$c'_k < c^*_k \rightarrow x_k[p_0(c^*_0), p_1(c^*_1), \ldots, p_k(c'_k), \ldots, p_n(c^*_n)] = 1$, by condition (1,n)-NL. Therefore:
$(2) \wedge c'_k < c^*_k \rightarrow c'_k + L_k(c^*_0, c'_k) < c^*_k + x^*_k L_k(c^*_0, c^*_k)$, as $x_k[p_0(c^*_0), p_1(c^*_1), \ldots, p_k(c^*_k), \ldots, p_n(c^*_n)] = x^*_k$
$\rightarrow c'_k + L_k(c^*_0, c'_k) < c^*_k + L_k(c^*_0, c^*_k)$, as $0 \leq x^*_k \leq 1$.
Adding $\Sigma_{i \in N-\{k\}} c^*_i + \Sigma_{i \in N_V-\{k\}} L_i(c^*_0, c^*_i)$ to both sides we obtain:
$TSC(c^*_0, c^*_1, \ldots, c'_k, \ldots, c^*_n) < TSC(c^*_0, c^*_1, \ldots, c^*_k, \ldots, c^*_n)$, a contradiction. (5)

Finally suppose (2) holds and $c'_k > c^*_k, k \in N_V$.
$c'_k > c^*_k \rightarrow x_k[p_0(c^*_0), p_1(c^*_1), \ldots, p_k(c'_k), \ldots, p_n(c^*_n)] = x_k[p_0(c^*_0), p_1(c^*_1), \ldots, p_k(c^*_k), \ldots, p_n(c^*_n)] = x^*_k$.
Therefore:
$(2) \wedge c'_k > c^*_k \rightarrow (1 - x^*_k)c'_k + x^*_k[c'_k + L_k(c^*_0, c'_k)] < (1 - x^*_k)c^*_k + x^*_k[c^*_k + L_k(c^*_0, c^*_k)]$.
Adding $\Sigma_{i \in N-\{k\}} x^*_k c^*_i + \Sigma_{i \in N_V-\{k\}} x^*_k L_i(c^*_0, c^*_i)$ to both sides we obtain:
$(1 - x^*_k)c'_k + x^*_k TSC(c^*_0, c^*_1, \ldots, c'_k, \ldots, c^*_n) < (1 - x^*_k)c^*_k + x^*_k TSC(c^*_0, c^*_1, \ldots, c^*_k, \ldots, c^*_n)$.
$\rightarrow (1 - x^*_k)c'_k < (1 - x^*_k)c^*_k$, as $TSC(c^*_0, c^*_1, \ldots, c'_k, \ldots, c^*_n) \geq TSC(c^*_0, c^*_1, \ldots, c^*_k, \ldots, c^*_n)$ and $x^*_k \geq 0$.
$(1 - x^*_k)c'_k < (1 - x^*_k)c^*_k \rightarrow 0 < 0$, if $(1 - x^*_k) = 0$; a contradiction. (6)
$(1 - x^*_k)c'_k < (1 - x^*_k)c^*_k \rightarrow c'_k < c^*_k$, if $(1 - x^*_k) > 0$; contradicting the hypothesis that $c'_k > c^*_k$. (7)
(6) and (7) establish that (2) cannot hold with $c'_k > c^*_k, k \in N_V$. (8)
(3), (4), (5) and (8) establish the lemma.

Lemma 2 *Let (1,n)-liability rule f satisfy the condition of (1,n)-negligence liability. Let $< C_0, C_1, \ldots, C_n; \mathcal{L}_1, \ldots, \mathcal{L}_n; (c^*_0, c^*_1, \ldots, c^*_n) \in M >$ be an application belonging to $\mathcal{A}'$. Then, $(\forall (\bar{c}_0, \bar{c}_1, \ldots, \bar{c}_n) \in C_0 \times C_1 \times \ldots \times C_n)[(\bar{c}_0, \bar{c}_1, \ldots, \bar{c}_n)$ is a Nash equilibrium $\rightarrow (\bar{c}_0, \bar{c}_1, \ldots, \bar{c}_n) \in M]$.*

Proof: Let (1,n)-liability rule f satisfy the condition of (1,n)-negligence liability; and let $< C_0, C_1, \ldots, C_n; \mathcal{L}_1, \ldots, \mathcal{L}_n; (c^*_0, c^*_1, \ldots, c^*_n) \in M >$ be an application belonging to $\mathcal{A}'$. Let $(\bar{c}_0, \bar{c}_1, \ldots, \bar{c}_n)$ be a Nash equilibrium.
$(\bar{c}_0, \bar{c}_1, \ldots, \bar{c}_n)$ being a Nash equilibrium implies:
$(\forall c_0 \in C_0)[\bar{c}_0 + \Sigma_{i \in N_V} y_i[p_0(\bar{c}_0), p_1(\bar{c}_1), \ldots, p_n(\bar{c}_n)] L_i(\bar{c}_0, \bar{c}_i) \leq$
$c_0 + \Sigma_{i \in N_V} y_i[p_0(c_0), p_1(\bar{c}_1), \ldots, p_n(\bar{c}_n)] L_i(c_0, \bar{c}_i)]$; and (1)

$$(\forall i \in N_V)(\forall c_i \in C_i)[\overline{c}_i + x_i[p_0(\overline{c}_0), p_1(\overline{c}_1), \ldots, p_i(\overline{c}_i), \ldots, p_n(\overline{c}_n)]L_i(\overline{c}_0, \overline{c}_i) \leq c_i + x_i[p_0(\overline{c}_0), p_1(\overline{c}_1), \ldots, p_i(c_i), \ldots, p_n(\overline{c}_n)]L_i(\overline{c}_0, c_i)]. \quad (2)$$

$$(1) \rightarrow [\overline{c}_0 + \Sigma_{i\in N_V} y_i[p_0(\overline{c}_0), p_1(\overline{c}_1), \ldots, p_n(\overline{c}_n)]L_i(\overline{c}_0, \overline{c}_i) \leq c_0^* + \Sigma_{i\in N_V} y_i[p_0(c_0^*), p_1(\overline{c}_1), \ldots, p_n(\overline{c}_n)]L_i(c_0^*, \overline{c}_i)]. \quad (3)$$

$$(2) \rightarrow (\forall i \in N_V)[\overline{c}_i + x_i[p_0(\overline{c}_0), p_1(\overline{c}_1), \ldots, p_i(\overline{c}_i), \ldots, p_n(\overline{c}_n)]L_i(\overline{c}_0, \overline{c}_i) \leq c_i^* + x_i[p_0(\overline{c}_0), p_1(\overline{c}_1), \ldots, p_i(c_i^*), \ldots, p_n(\overline{c}_n)]L_i(\overline{c}_0, c_i^*)]. \quad (4)$$

(3) and (4) $\rightarrow$ $\Sigma_{i\in N}\overline{c}_i + \Sigma_{i\in N_V} L_i(\overline{c}_0, \overline{c}_i) \leq \Sigma_{i\in N} c_i^* + \Sigma_{i\in N_V} x_i[(\forall j \in N - \{i\})(p_j = p_j(\overline{c}_j)) \wedge p_i = p_i(c_i^*)]L_i[\overline{c}_0, c_i^*] + \Sigma_{i\in N_V} y_i[(\forall j \in N_V)(p_j = p_j(\overline{c}_j)) \wedge p_0 = p_0(c_0^*)]$
$L_i[c_0^*, \overline{c}_i]$. (5)

For $i \in N_V$ we have:
$\overline{c}_0 < c_0^* \wedge \overline{c}_i < c_i^* \rightarrow x_i[(\forall j \in N - \{i\})(p_j = p_j(\overline{c}_j)) \wedge p_i = p_i(c_i^*)] = 0 \wedge y_i[(\forall j \in N_V)(p_j = p_j(\overline{c}_j)) \wedge p_0 = p_0(c_0^*)] = 0$, by condition (1,n)-NL
Therefore, $x_i[(\forall j \in N - \{i\})(p_j = p_j(\overline{c}_j)) \wedge p_i = p_i(c_i^*)]L_i[\overline{c}_0, c_i^*] + y_i[(\forall j \in N_V)(p_j = p_j(\overline{c}_j)) \wedge p_0 = p_0(c_0^*)]L_i[c_0^*, \overline{c}_i] = 0$ (6)

$\overline{c}_0 < c_0^* \wedge \overline{c}_i \geq c_i^* \rightarrow x_i[(\forall j \in N - \{i\})(p_j = p_j(\overline{c}_j)) \wedge p_i = p_i(c_i^*)] = 0$ by (1,n)-NL
Therefore, $x_i[(\forall j \in N - \{i\})(p_j = p_j(\overline{c}_j)) \wedge p_i = p_i(c_i^*)]L_i[\overline{c}_0, c_i^*] + y_i[(\forall j \in N_V)(p_j = p_j(\overline{c}_j)) \wedge p_0 = p_0(c_0^*)]L_i[c_0^*, \overline{c}_i] = y_i[(\forall j \in N_V)(p_j = p_j(\overline{c}_j)) \wedge p_0 = p_0(c_0^*)]L_i[c_0^*, \overline{c}_i]$
$\leq L_i[c_0^*, \overline{c}_i]$
$\leq L_i[c_0^*, c_i^*]$ by Assumptions (A4) and (A5) (7)

$\overline{c}_0 \geq c_0^* \wedge \overline{c}_i < c_i^* \rightarrow y_i[(\forall j \in N_V)(p_j = p_j(\overline{c}_j)) \wedge p_0 = p_0(c_0^*)] = 0$, by condition (1,n)-NL
Therefore, $x_i[(\forall j \in N - \{i\})(p_j = p_j(\overline{c}_j)) \wedge p_i = p_i(c_i^*)]L_i[\overline{c}_0, c_i^*] + y_i[(\forall j \in N_V)(p_j = p_j(\overline{c}_j)) \wedge p_0 = p_0(c_0^*)]L_i[c_0^*, \overline{c}_i] = x_i[(\forall j \in N - \{i\})(p_j = p_j(\overline{c}_j)) \wedge p_i = p_i(c_i^*)]L_i[\overline{c}_0, c_i^*]$
$\leq L_i[\overline{c}_0, c_i^*]$
$\leq L_i[c_0^*, c_i^*]$ by Assumptions (A4) and (A5) (8)

$\overline{c}_0 \geq c_0^* \wedge \overline{c}_i \geq c_i^* \rightarrow [(\forall j \in N - \{i\})(p_j = p_j(\overline{c}_j)) \wedge p_i = p_i(c_i^*)] = [p_0 = 1, p_1(\overline{c}_1), \ldots, p_i = 1, \ldots, p_n(\overline{c}_n)] = [(\forall j \in N_V)(p_j = p_j(\overline{c}_j)) \wedge p_0 = p_0(c_0^*)]$
Therefore, $x_i[(\forall j \in N - \{i\})(p_j = p_j(\overline{c}_j)) \wedge p_i = p_i(c_i^*)]L_i[\overline{c}_0, c_i^*] + y_i[(\forall j \in N_V)(p_j = p_j(\overline{c}_j)) \wedge p_0 = p_0(c_0^*)]L_i[c_0^*, \overline{c}_i] \leq x_i[(\forall j \in N - \{i\})(p_j = p_j(\overline{c}_j)) \wedge p_i = p_i(c_i^*)]L_i[c_0^*, c_i^*] +$

$y_i[(\forall j \in N_V)(p_j = p_j(\overline{c}_j)) \ \wedge \ p_0 = p_0(c_0^*)]L_i[c_0^*, c_i^*]$, by Assumptions (A4) and (A5)
$= L_i[c_0^*, c_i^*]$. (9)

In view of (6)-(9), (5) implies:
$\Sigma_{i \in N}\overline{c}_i + \Sigma_{i \in N_V} L_i(\overline{c}_0, \overline{c}_i) \leq \Sigma_{i \in N} c_i^* + \Sigma_{i \in N_V} L_i[c_0^*, c_i^*]$
$\rightarrow TSC(\overline{c}_0, \overline{c}_1, \ldots, \overline{c}_n) \leq TSC(c_0^*, c_1^*, \ldots, c_n^*)$.
As TSC is minimum at $(c_0^*, c_1^*, \ldots, c_n^*)$, we in fact must have: $TSC(\overline{c}_0, \overline{c}_1, \ldots, \overline{c}_n) = TSC(c_0^*, c_1^*, \ldots, c_n^*)$, which implies that $(\overline{c}_0, \overline{c}_1, \ldots, \overline{c}_n) \in M$. This concludes the proof.

Theorem 2 *If (1,n)-liability rule f satisfies the condition of (1,n)-negligence liability then it is efficient with respect to $\mathcal{A}'$.*

Proof: Let (1,n)-liability rule f satisfy the condition of (1,n)-negligence liability. Let $< C_0, C_1, \ldots, C_n; \mathcal{L}_1, \ldots, \mathcal{L}_n; (c_0^*, c_1^*, \ldots, c_n^*) \in M >$ be any application belonging to $\mathcal{A}'$. Then, $(c_0^*, c_1^*, \ldots, c_n^*)$ is a Nash equilibrium by Lemma 1, and we have: $(\forall(\overline{c}_0, \overline{c}_1, \ldots, \overline{c}_n) \in C_0 \times C_1 \times \ldots \times C_n)[(\overline{c}_0, \overline{c}_1, \ldots, \overline{c}_n)$ is a Nash equilibrium $\rightarrow$ $(\overline{c}_0, \overline{c}_1, \ldots, \overline{c}_n) \in M]$ by Lemma 2; establishing efficiency of f with respect to $\mathcal{A}'$.

By Theorem 2, the condition of (1,n)-negligence liability is a sufficient condition for any (1,n)-liability rule to be efficient with respect to $\mathcal{A}'$. Applications belonging to $\mathcal{A}'$ have the characteristic that expected loss of a particular victim depends only on his own care level and the care level of the injurer. When expected loss of a victim depends only on his own care level and the care level of the injurer, the use of a (1,n)-liability rule belonging to $\mathcal{F}'$ seems particulary appropriate, where the proportion of loss that a particular victim has to bear depends only on the nonnegligence proportions of the injurer and the victim in question. The next theorem shows that if we consider the subclass $\mathcal{F}'$ of (1,n)-liability rules then the condition of (1,n)-negligence liability is both necessary and sufficient for efficiency with respect to $\mathcal{A}'$. In other words, the rules in $\mathcal{F}'$ which are efficient with respect to $\mathcal{A}'$ are characterized by the condition of (1,n)-negligence liability. Whether (1,n)-negligence liability is necessary for any (1,n)-liability rule belonging to $\mathcal{F}$ to be efficient with respect to $\mathcal{A}'$ is an open question.

Theorem 3 *Let f be a (1,n)-liability rule belonging to $\mathcal{F}'$. Then f is efficient for every application belonging to $\mathcal{A}'$ iff it satisfies the condition of (1,n)-negligence liability.*

Proof: Let (1,n)-liability rule f belong to $\mathcal{F}'$. Suppose f violates (1,n)-NL. Then we must have:
$(\exists k \in N_V)(\exists (p_0, p_1, \ldots, p_n) \in [0,1]^{n+1})[[p_0 < 1 \wedge p_k = 1 \wedge x_k(p_0, p_1, \ldots, p_n) > 0] \vee [p_0 = 1 \wedge p_k < 1 \wedge y_k(p_0, p_1, \ldots, p_n) > 0]]$.

First suppose $(\exists k \in N_V)(\exists (p_0, p_1, \ldots, p_n) \in [0,1]^{n+1})[p_0 < 1 \wedge p_k = 1 \wedge x_k(p_0, p_1, \ldots, p_n) > 0]$.
Let this $p_0 < 1$ be designated by $\overline{p}_0$. As f belongs to $\mathcal{F}'$, it follows that: we must have $x_k(p_0 = \overline{p}_0, (\forall i \in N_V)(p_i = 1)] = \overline{x}_k > 0$.

Now consider the application belonging to $\mathcal{A}'$ specified below:
Let $t > 0$.
Choose r_1, r_2 such that $(1 - \overline{x}_k)t = \overline{y}_k t < r_1 < r_2 < t$.
Let $\overline{c}_0 = \frac{r_2}{1-\overline{p}_0}$.
Let $(\forall j \in N_V)(\overline{c}_j > 0 \wedge \delta_j > 0)$.
Choose ϵ such that: $0 < \epsilon < \frac{r_2 - r_1}{n-1}$.
Let: $C_0 = \{0, \overline{p}_0\overline{c}_0, \overline{c}_0\}; C_k = \{0, \overline{c}_k\}; (\forall i \in N_V - \{k\})[C_i = \{0, \overline{c}_i\}]$.
Let $L_k(c_0, c_k)$ and $L_i(c_0, c_i), i \in N_V - \{k\}$, be as specified in the following arrays respectively.[6]

		c_k: 0	c_k: $\overline{c}_k$
c_0	0	$\overline{c}_k + \delta_k + \frac{\overline{p}_0\overline{c}_0}{n} + t$	$\frac{\overline{p}_0\overline{c}_0}{n} + t$
	$\overline{p}_0\overline{c}_0$	$\overline{c}_k + \delta_k + t$	t
	$\overline{c}_0$	$\overline{c}_k + \delta_k$	0

		c_i: 0	c_i: $\overline{c}_i$
c_0	0	$\overline{c}_i + \delta_i + \frac{\overline{p}_0\overline{c}_0}{n} + \epsilon$	$\frac{\overline{p}_0\overline{c}_0}{n} + \epsilon$
	$\overline{p}_0\overline{c}_0$	$\overline{c}_i + \delta_i + \epsilon$	ϵ
	$\overline{c}_0$	$\overline{c}_i + \delta_i$	0

$(\forall j \in N_V)(\delta_j > 0)$ and $t > r_2$ imply that $(\forall i \in N)(c_i = \overline{c}_i)$ is the unique total social cost

[6] Specifications of $L_k(c_0, c_k)$ and $L_i(c_0, c_i), i \in N_V - \{k\}$, are done in such a way that no inconsistency would arise even if $\overline{p}_0 = 0$.

minimizing configuration of care levels.
Let $(c_0^*, c_1^*, \ldots, c_n^*) = (\forall i \in N)(c_i = \overline{c}_i)$.

Now, given that every $i \in N, i \neq 0$, is using $c_i = c_i^*$;
If the injurer uses $c_0 = \overline{c}_0$, then his expected costs $= EC_0(\overline{c}_0, \overline{c}_1, \ldots, \overline{c}_n) = \overline{c}_0$, as $(\forall i \in N_V)(L_i(\overline{c}_0, \overline{c}_i) = 0)$
If the injurer uses $c_0 = \overline{p}_0\overline{c}_0$, then his expected costs $= EC_0(\overline{p}_0\overline{c}_0, \overline{c}_1, \ldots, \overline{c}_n) = \overline{p}_0\overline{c}_0 + \overline{y}_k t + \Sigma_{i \in N_V - \{k\}} y_i(\overline{p}_0, 1, \ldots, 1)\epsilon$.
$EC_0(\overline{c}_0, \overline{c}_1, \ldots, \overline{c}_n) - EC_0(\overline{p}_0\overline{c}_0, \overline{c}_1, \ldots, \overline{c}_n) = \overline{c}_0 - [\overline{p}_0\overline{c}_0 + \overline{y}_k t + \Sigma_{i \in N_V - \{k\}} y_i(\overline{p}_0, 1, \ldots, 1)\epsilon]$
$\geq (1 - \overline{p}_0)\overline{c}_0 - \overline{y}_k t - (n-1)\epsilon$
$> (1 - \overline{p}_0)\overline{c}_0 - \overline{y}_k t - (r_2 - r_1)$
$= r_1 - \overline{y}_k t$
> 0.

Thus, given that every $i \in N, i \neq 0$, is using $c_i = c_i^*$, for the injurer $c_0 = \overline{p}_0\overline{c}_0$ is better than $c_0 = \overline{c}_0$. Thus the only TSC-minimizing configuration is not a Nash equilibrium. This establishes that f is not efficient with respect to $\mathcal{A}'$. (1)

Next suppose that: $(\exists k \in N_V)(\exists (p_0, p_1, \ldots, p_n) \in [0,1]^{n+1})[p_0 = 1 \wedge p_k < 1 \wedge y_k(p_0, p_1, \ldots, p_n) > 0]$.
Let this $p_k < 1$ be designated by $\overline{p}_k$. As f belongs to $\mathcal{F}'$, it follows that: we must have $y_k[(\forall i \in N - \{k\})(p_i = 1) \wedge p_k = \overline{p}_k] = \overline{y}_k > 0$.

Now consider the application belonging to $\mathcal{A}'$ specified below:
Let $t > 0$.
Choose r such that $(1 - \overline{y}_k)t = \overline{x}_k t < r < t$.
Let $\overline{c}_k = \frac{r}{1 - \overline{p}_k}$.
Let $(\forall i \in N_V - \{k\})(\overline{c}_i > 0 \wedge \delta_i > 0)$.
Choose $\overline{c}_0$ and ϵ such that: $0 < \frac{\overline{c}_0}{n} < \epsilon$.
Let: $C_0 = \{0, \overline{c}_0\}; (\forall i \in N_V - \{k\})(C_i = \{0, \overline{c}_i\}); C_k = \{0, \overline{p}_k\overline{c}_k, \overline{c}_k\}$.
Let $L_k(c_0, c_k)$ and $L_i(c_0, c_i), i \in N_V - \{k\}$, be as specified in the following arrays respectively.[7]

[7]Specification of $L_k(c_0, c_k)$ is done in such a way that no inconsistency would arise even if $\overline{p}_k = 0$.

		c_k		
		0	$\overline{p}_k\overline{c}_k$	$\overline{c}_k$
c_0	0	$\overline{p}_k\overline{c}_k + t + \epsilon$	$t + \epsilon$	ϵ
	$\overline{c}_0$	$\overline{p}_k\overline{c}_k + t$	t	0

		c_i	
		0	$\overline{c}_i$
c_0	0	$\overline{c}_i + \delta_i + \epsilon$	ϵ
	$\overline{c}_0$	$\overline{c}_i + \delta_i$	0

$\overline{c}_0 < n\epsilon$, $(\forall i \in N_V - \{k\})(\delta_i > 0)$ and $t > r$ imply that $(\forall i \in N)(c_i = \overline{c}_i)$ is the unique total social cost minimizing configuration of care levels.
Let $(c_0^*, c_1^*, \ldots, c_n^*) = (\forall i \in N)(c_i = \overline{c}_i)$.

Now, given that every $i \in N, i \neq k$, is using $c_i = c_i^*$;
If victim k uses $c_k = \overline{c}_k$, then his expected costs $= EC_k(\overline{c}_0, \overline{c}_1, \ldots, \overline{c}_n) = \overline{c}_k$, as $L_k(\overline{c}_0, \overline{c}_k) = 0$
If victim k uses $c_k = \overline{p}_k\overline{c}_k$, then his expected costs $= EC_k(\overline{c}_0, \overline{c}_1, \ldots, \overline{p}_k\overline{c}_k, \ldots, \overline{c}_n) = \overline{p}_k\overline{c}_k + \overline{x}_k t$.
$EC_k(\overline{c}_0, \overline{c}_1, \ldots, \overline{c}_k, \ldots, \overline{c}_n) - EC_k(\overline{c}_0, \overline{c}_1, \ldots, \overline{p}_k\overline{c}_k, \ldots, \overline{c}_n) = \overline{c}_k - \overline{p}_k\overline{c}_k - \overline{x}_k t$
$= (1 - \overline{p}_k)\overline{c}_k - \overline{x}_k t$
$= r - \overline{x}_k t$
> 0.

Thus, given that every $i \in N, i \neq k$, is using $c_i = c_i^*$, for victim k $c_k = \overline{p}_k\overline{c}_k$ is better than $c_k = \overline{c}_k$. Thus the only TSC minimizing configuration of care levels is not a Nash equilibrium. This establishes that f is not efficient with respect to $\mathcal{A}'$. (2)

(1) and (2) establish the necessity of (1,n)-negligence liability for a rule belonging to $\mathcal{F}'$ to be efficient with respect to $\mathcal{A}'$. The sufficiency of (1,n)-negligence liability for a rule belonging to $\mathcal{F}'$ to be efficient with respect to $\mathcal{A}'$ follows from Theorem 2.

4 Concluding Remarks

In the context of one victim and multiple injurers there exist liability rules which are efficient for all applications. On the other hand, as shown in this paper, there are no liability rules which are efficient for all applications when there are multiple victims and one injurer. The reason for this difference lies in the fact that for efficiency what is required is that all parties involved internalize the totality of harm resulting from the interaction. This is possible when there are multiple injurers and one victim; but not when there are multiple victims and one injurer. Regardless of how much care is taken by a victim, he can at most be made to bear his own loss in entirety; but not any part of loss incurred by another victim. There are contexts in which the loss that a particular victim suffers depends not only on the care taken by himself and the injurer but also on the care levels of other victims. In such situations, given that the victims can at most be made to bear their own losses in entirety, there is no way that a victim could be made to internalize the loss incurred by another victim. Consequently, unlike the case of one victim and multiple injurers, in the case of one injurer and multiple victims one gets an impossibility theorem. If one considers only those applications where expected loss of a victim depends only on his own care level and the care level of the injurer, but not on the care level of another victim, then it becomes possible to make all parties internalize all losses which they are in a position to affect; and one obtains possibility theorems.

It is of some interest to note that, like the condition of collective negligence liability[8], the condition of (1,n)-negligence liability can also be regarded as a generalization of the condition of negligence liability[9]. As is the case with the condition of collective negligence liability, the condition of (1,n)-negligence liability also reduces to that of negligence liability when there are only two parties, one victim and one injurer.

[8]A liability rule defined for one victim (individual 1) and multiple injurers (individuals $2, \ldots, n+1$) satisfies the condition of collective negligence liability iff $(\forall (p_1, \ldots, p_{n+1}) \in [0,1]^{n+1})[(p_1, \ldots, p_{n+1}) \neq (1, \ldots, 1) \rightarrow (\forall i \in \{1, 2, \ldots, n+1\})(p_i = 1 \rightarrow x_i(p_1, \ldots, p_{n+1}) = 0)]$, where $x_i, i \in \{1, 2, \ldots, n+1\}$, denotes the share of the loss incurred by the single victim to be borne by individual i; and $\Sigma_{i \in \{1,2,\ldots,n+1\}} x_i = 1$.

[9]A liability rule defined for one victim (individual 1) and one injurer (individual 2) satisfies the condition of negligence liability iff $(\forall (p_1, p_2) \in [0,1]^2)[[p_1 < 1 \wedge p_2 = 1 \rightarrow x_1(p_1, p_2) = 1] \wedge [p_1 = 1 \wedge p_2 < 1 \rightarrow x_2(p_1, p_2) = 1]]$, where $x_i, i \in \{1, 2\}$, denotes the share of the loss incurred by individual 1, the victim, to be borne by individual i; and $\Sigma_{i \in \{1,2\}} x_i = 1$.

From the results of this paper it is clear that there does not exist any liability rule defined for multiple injurers and multiple victims which is efficient for all applications. However, from the results which have been obtained regarding the efficiency of liability rules defined for one victim and multiple injurers and the results of this paper, it appears that, if efficiency is considered with respect to $\mathcal{A}'$, then possibility results should obtain for multi-injurer multi-victim liability rules.

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