On the Efficiency of Negligence Rule*

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Abstract

In the law and economics literature there are three different versions of negligence rule which have been discussed. These three versions are: (i) Injurer is liable for the entire loss if negligent, and not liable if nonnegligent. Injurer is negligent if his care level is below the due care level, otherwise nonnegligent. (ii) Injurer is liable for the incremental loss if negligent, and not liable if nonnegligent. Injurer is negligent if his care level is below the due care level, otherwise nonnegligent. (iii) Injurer is liable for the incremental loss if negligent, and not liable if nonnegligent. Injurer is negligent if there exists a precaution which could have been taken but was not, and which would have brought about reduction in expected loss of a magnitude greater than the cost of precaution; otherwise nonnegligent. In the literature it is taken for granted that all three versions of negligence rule are efficient. A careful analysis, however, shows that version (iii) is not efficient. This version, in fact, is not efficient even for the unilateral case. Efficiency of version (i) was established by Brown. Efficiency of version (ii) for the unilateral case was shown by Kahan; efficiency for the bilateral case is established in this paper.

Keywords: Standard Negligence Rule, Incremental Negligence Rule, Negligence as Shortfall from Due Care, Negligence as Existence of a Cost-Justified Untaken Precaution, Efficiency, Strategic Manipulability

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1 Introduction

In the law and economics literature there are three versions of negligence rule which have been discussed. These three versions are: (i) Injurer is liable for the entire loss if he is negligent; and he is not at all liable if he is nonnegligent. An injurer is negligent if his care level is less than a specified level of care, called due care; otherwise he is nonnegligent. This version of negligence rule will be refereed to as the standard version with negligence defined as shortfall from due care. (ii) When injurer is nonnegligent he is not liable at all, and when he is negligent he is liable for that amount of loss which can be attributed to his negligence; the notion of negligence being defined as in the first case. This version of negligence rule will be referred to as the incremental version with negligence defined as shortfall from due care. (iii) As in the previous case, if nonnegligent, injurer is not liable; and if negligent, he is liable for the loss which can be attributed to his negligence. Injurer is negligent if there exists a precaution which he could have taken but did not, and which would have cost less than the reduction it would have brought about in the expected loss. And injurer is nonnegligent if there does not exist any such cost-justified untaken precaution. This version will be refereed to as the incremental version with negligence defined as existence of a cost-justified untaken precaution.

The different versions of negligence rule arise because of differences in the two key ideas. One idea relates to the liability of a negligent injurer. In the context of negligence rule a negligent injurer can be made liable for the entire loss or only that part of the loss which is attributable to his negligence. Negligence rule under which a negligent injurer is made liable for the entire loss can be referred to as the standard negligence rule; and the one under which a negligent injurer is made liable only for the loss which can be attributed to his negligence as the incremental negligence rule. The second idea relates to what constitutes negligence. In the law and economics literature there are two different ways in which the notion of negligence has been defined. The mainstream approach of defining negligence consists of specifying a care level, called the due care level; and declaring an injurer to be negligent if his care level is below the due care level, and nonnegligent otherwise. There is, however, another way of defining the notion of negligence which is there in the law and economics literature, based on analysis of costs and benefits. If there exists an untaken precaution taking of which would have cost less than the reduction in expected loss that it would have brought about then from a certain perspective the conduct of the party not taking the cost-justified precaution can be viewed as negligent. Accordingly from this perspective, an injurer is negligent if there exists a cost-justified untaken precaution; and he is nonnegligent if there does not exist any cost-justified untaken precaution.[1]

As one can use either notion of negligence with each of the two ways, standard and incremental, of defining the negligence rule, one obtains four versions of negligence rule. In addition to the three versions listed above one can also consider the standard negligence rule with negligence defined as existence of a cost-justified untaken precaution. This version, however, appears not to have been discussed in the literature.

The efficiency of the standard negligence rule with negligence defined as shortfall from due care was established by Brown (1973). He showed that, when care is bilateral, under this version of the negligence rule both victim and injurer are led to take socially optimal levels of care. As, a rule efficient under bilateral care is also efficient under unilateral care, but not necessarily the other way round, from the Brown result it follows that the standard negligence rule with negligence defined as shortfall from due care is efficient under unilateral as well as bilateral care. The incremental version of the rule with negligence as shortfall from due care was analyzed by Kahan (1989). He showed that, when care is unilateral, incremental negligence rule with negligence defined as shortfall from due care is an efficient rule. We will show in this paper that this version of negligence rule is efficient under bilateral care as well, from which Kahan result will follow as a corollary. Unlike the first two versions, the third version of negligence rule does not turn out to be efficient. It is shown in the paper that incremental negligence rule with negligence defined as existence of a cost-justified untaken precaution is not an efficient rule regardless of whether care is bilateral or unilateral.

The paper is organized in six sections. The section following this introduction contains the framework within which efficiency of negligence rule is analyzed here. Section 3 contains the proof that incremental negligence rule with negligence defined as shortfall from due care is efficient under bilateral care. In section 4 it is shown that incremental negligence rule with negligence defined as existence of a cost-justified untaken precaution is inefficient when care is bilateral. Section 5 analyzes the efficiency of this version when care is unilateral. It is shown there this version continues to be inefficient even under unilateral care. To render this version efficient under unilateral care further restrictions are required. It is argued there that conditions which would ensure efficiency of incremental negligence rule with negligence defined as existence of a cost-justified untaken precaution under unilateral care are likely to be highly restrictive. The last section contains some remarks on the implications of incremental negligence rule with negligence defined as existence of a cost-justified untaken precaution turning out to be an inefficient rule.

[1] This way of defining negligence has been pioneered by Grady (1983, 1984, 1989).

2 Definitions and Assumptions

We consider accidents resulting from interaction of two parties, assumed to be strangers to each other, in which, to begin with, the entire loss falls on one party to be called the victim (plaintiff). The other party would be referred to as the injurer (defendant). At times, the victim would be refereed to as individual or party 1 and the injurer individual or party 2. We denote by $c \geq 0$ the cost of care taken by the victim and by $d \geq 0$ the cost of care taken by the injurer. Costs of care would be assumed to be strictly increasing functions of indices of care, i.e., care levels; consequently, costs of care themselves can be taken to be indices of care. Let

$C = \{c \mid c$ is the cost of some feasible level of care which can be taken by the victim$\}$

and

$D = \{d \mid d$ is the cost of some feasible level of care which can be taken by the injurer$\}$.

We will identify $c = 0$ with victim taking no care; and $d = 0$ with injurer taking no care.

We assume:

$$0 \in C \ \wedge \ 0 \in D. \tag{A1}$$

Assumption (A1) merely says that, for each party, taking no care is always a feasible option.

Let π denote the probability of occurrence of accident and $H \geq 0$ the loss in case of occurrence of accident. Both π and H will be assumed to be functions of c and d; $\pi = \pi(c, d), H = H(c, d)$. Let $L = \pi H$. L is thus expected loss due to accident.

We assume:

$$(\forall c, c' \in C)(\forall d, d' \in D)[[c > c' \ \rightarrow \ \pi(c, d) \leq \pi(c', d)] \ \wedge \ [d > d' \ \rightarrow \ \pi(c, d) \leq \pi(c, d')]]. \tag{A2}$$

and

$$(\forall c, c' \in C)(\forall d, d' \in D)[[c > c' \ \rightarrow \ H(c, d) \leq H(c', d)] \ \wedge \ [d > d' \ \rightarrow \ H(c, d) \leq H(c, d')]]. \tag{A3}$$

In other words, it is assumed that a larger expenditure on care by either party, given the expenditure on care by the other party, does not result in higher probability of occurrence of accident or in larger accident loss.

From (A2) and (A3) it follows that:

$$(\forall c, c' \in C)(\forall d, d' \in D)[[c > c' \ \rightarrow \ L(c, d) \leq L(c', d)] \ \wedge \ [d > d' \ \rightarrow \ L(c, d) \leq L(c, d')]].$$

That is to say: a larger expenditure on care by either party, given the expenditure on care by the other party, results in lesser or equal expected accident loss.

Total social costs (TSC) are defined to be the sum of cost of care by the victim, cost of care by the injurer, and expected loss due to accident; $TSC = c + d + L(c, d)$. Let $M = \{(c', d') \in C \times D \mid c' + d' + L(c', d')$ is minimum of $\{c + d + L(c, d) \mid c \in C \wedge d \in D\}\}$. Thus M is the set of all costs of care configurations (c', d') which are total social cost minimizing. It will be assumed that:
C, D and L are such that M is nonempty. (A4)

The notion of negligence is usually defined as shortfall (sf) from a specified level called due care. Let d^* denote the due care for the injurer. It will be assumed that: $(\exists c^* \in C)[(c^*, d^*) \in M]$. That is to say, due care for the injurer is chosen appropriately from the perspective of minimization of total social costs. When the notion of negligence is defined in terms of shortfall from due care, the injurer is called negligent at (c, d) iff his care level d is less than d^*; and nonnegligent iff d is greater than or equal to d^*.

Another way to conceptualize negligence is in terms of cost-justified untaken precautions (up).
Corresponding to each $(c, d) \in C \times D$, we define:
$D^u(c, d) = \{d^u \in D \mid d^u > d \wedge L(c, d) - L(c, d^u) > d^u - d\}$.
Thus, $D^u(c, d)$ is the set of all cost-justified untaken precautions at (c, d) which the injurer could have taken. When the notion of negligence is defined in terms of cost-justified untaken precautions the injurer is called negligent at (c, d) iff $D^u(c, d)$ is nonempty; and nonnegligent at (c, d) iff $D^u(c, d)$ is empty. In other words, at (c, d), the injurer is defined to be negligent iff there is a cost-justified untaken precaution; and nonnegligent iff there does not exist any cost-justified untaken precaution.

There are two versions of negligence rule which are used in practice. The standard (s) version of negligence rule is defined by: (a) The injurer is liable for the entire loss iff he is negligent; and (b) The injurer is not at all liable iff he is nonnegligent. The incremental (i) version of negligence rule is defined by: (a) The injurer is liable for the loss which can be ascribed to his negligence iff he is negligent; and (b) The injurer is not at all liable iff he is nonnegligent. Each of these two versions can be considered in conjunction with either of the two notions of negligence defined above. The standard version with negligence as shortfall from due care, incremental version with negligence as shortfall from due care, the standard version with negligence as existence of a cost-justified untaken precaution, and the incremental version with negligence as existence of a cost-justified untaken precaution would be written in abbreviated form as (s-sf), (i-sf), (s-up), (i-up) respectively. In the law and economics literature only the versions (s-sf), (i-sf) and (i-up) have been discussed. In this paper also we we will discuss only these three versions.

Let $\hat{L}_2(c,d)$ denote expected loss which can be ascribed to injurer's negligence at (c,d). When negligence is defined in terms of shortfall from due care we define $\hat{L}_2(c,d)$ as follows:

$$\begin{aligned}\hat{L}_2(c,d) &= L(c,d) - L(c,d^*) \quad \text{if} \quad d < d^* \\ &= 0 \quad \text{if} \quad d \geq d^*.\end{aligned}$$

Let: $L^u(c,d) = \{L(c,d) - L(c,d^u) \mid d^u \in D^u(c,d)\}$.
When negligence is defined in terms of existence of untaken precautions, we define $\hat{L}_2(c,d)$ as follows:

$$\begin{aligned}\hat{L}_2(c,d) &= \sup L^u(c,d) \quad \text{if} \quad D^u(c,d) \neq \emptyset \\ &= 0 \quad \text{if} \quad D^u(c,d) = \emptyset.\end{aligned}$$

Let $EC_1(c,d)$ and $EC_2(c,d)$ denote expected costs of the victim and the injurer respectively. Then, under the standard negligence rule, with negligence defined as shortfall from due care, the expected costs are given by:

$$\begin{aligned}&EC_1(c,d) = c \wedge EC_2(c,d) = d + L(c,d) \quad \text{if} \quad d < d^* \\ &EC_1(c,d) = c + L(c,d) \wedge EC_2(c,d) = d \quad \text{if} \quad d \geq d^*.\end{aligned}$$

Under the incremental negligence rule, with negligence defined as shortfall from due care, the expected costs are given by:

$$\begin{aligned}&EC_1(c,d) = c + L(c,d^*) \wedge EC_2(c,d) = d + L(c,d) - L(c,d^*) \quad \text{if} \quad d < d^* \\ &EC_1(c,d) = c + L(c,d) \wedge EC_2(c,d) = d \quad \text{if} \quad d \geq d^*.\end{aligned}$$

Under the incremental negligence rule, with negligence defined as existence of a cost-justified untaken precaution, the expected costs are given by:

$$\begin{aligned}&EC_1(c,d) = c + L(c,d) - \sup L^u(c,d) \wedge EC_2(c,d) = d + \sup L^u(c,d) \quad \text{if} \quad D^u(c,d) \neq \emptyset \\ &EC_1(c,d) = c + L(c,d) \wedge EC_2(c,d) = d \quad \text{if} \quad D^u(c,d) = \emptyset.\end{aligned}$$

Both parties are assumed to prefer smaller expected costs to larger expected costs and be indifferent between alternatives with equal expected costs.

When negligence is defined as shortfall from due care, an application of negligence rule, standard or incremental, consists of specification of C, D, π, H and d^* satisfying (A1)-(A4), where d^* is such that $(\exists c^* \in C)[(c^*, d^*) \in M]$. The set of all applications will be denoted by $\mathcal{A}_{sf}$.

When negligence is defined as existence of a cost-justified untaken precaution, an application of negligence rule, standard or incremental, consists of specification of C, D, π and H satisfying (A1)-(A4). The set of all $< C, D, \pi, H >$ satisfying (A1)-(A4) will be denoted by $\mathcal{A}_{up}$. Let $\mathcal{A}_{up}^{o_1} \subset \mathcal{A}_{up}$ denote the subset of applications which are such that $(\forall(\overline{c}, \overline{d}) \in C \times D)[(\overline{c}, \overline{d}) \in M \ \rightarrow \overline{c} = 0]$. Thus $\mathcal{A}_{up}^{o_1}$ is the set of applications which are such that victim's optimal care is zero. Let $D_M = \{d \in D \mid (\exists c \in C)[(c, d) \in M]\}$. Let $\mathcal{A}_{up}^{m_2} \subset \mathcal{A}_{up}$ denote the set of applications for which $min\ D_M$ exists.

Negligence rule, standard or incremental, is defined to be efficient for a given application belonging to $\mathcal{A}_{sf}$ or $\mathcal{A}_{up}$, as the case may be, iff $(\forall(\overline{c}, \overline{d}) \in C \times D)[(\overline{c}, \overline{d})$ is a Nash equilibrium $\rightarrow$ $(\overline{c}, \overline{d}) \in M]$ and $(\exists(\overline{c}, \overline{d}) \in C \times D)[(\overline{c}, \overline{d})$ is a Nash equilibrium]. In other words, negligence rule is efficient for a given application iff (i) every $(\overline{c}, \overline{d}) \in C \times D$ which is a Nash equilibrium is total social cost minimizing, and (ii) there exists at least one $(\overline{c}, \overline{d}) \in C \times D$ which is a Nash equilibrium. Negligence rule is defined to be efficient with respect to a class of applications iff it is efficient for every application belonging to that class.

3 Negligence as Shortfall from Due Care and Efficiency of Standard and Incremental Versions of Negligence Rule

Proposition 1 *(Brown) If negligence is defined as shortfall from due care then the standard version of the negligence rule is efficient for every application belonging to $\mathcal{A}_{sf}$.*

Efficiency of negligence rule (s-sf) was established by Brown (1973) in his seminal contribution which dealt with the efficiency or inefficiency of some of the most important liability rules used in practice. In Jain and Singh (2002) it is shown that, if negligence is defined as shortfall from due care, then a standard liability rule[2] is efficient for all applications iff it satisfies the condition of negligence liability. The condition of negligence liability requires that (a) whenever the injurer is nonnegligent and the victim is negligent, the entire loss in case of accident must be borne by the victim, and (ii) whenever the victim is nonnegligent and the injurer is negligent, the entire loss in case of accident must be borne by the injurer. It is immediate that negligence rule (s-sf) satisfies the condition of negligence liability. Thus the efficiency of negligence rule (s-sf) can also be deduced as a corollary of the theorem stating that efficiency is characterized by the condition of negligence liability in the context of standard liability rules when negligence is defined as

[2] A standard liability rule determines the proportions in which loss, in case of accident, is to be apportioned between the two parties on the basis of which parties are negligent and to what extent.

shortfall from due care.

We now consider the efficiency of incremental negligence rule when negligence is defined as shortfall from due care.

Lemma 1 *Let negligence be defined as shortfall from due care. Let* $< C, D, \pi, H, d^*) >$ *be an application belonging to* $\mathcal{A}_{sf}$*. Let* c^* *be such that* $(c^*, d^*) \in M$*. Then under the incremental negligence rule* (c^*, d^*) *is a Nash equilibrium.*

Proof: Let $< C, D, \pi, H, d^* > \in \mathcal{A}_{sf}$; and $(c^*, d^*) \in M$.

$EC_1(c^*, d^*) = c^* + L(c^*, d^*)$
$c \neq c^* \rightarrow EC_1(c, d^*) = c + L(c, d^*)$
Therefore, $c \neq c^* \rightarrow EC_1(c, d^*) - EC_1(c^*, d^*) = [c + L(c, d^*)] - [c^* + L(c^*, d^*)] = [c + d^* + L(c, d^*)] - [c^* + d^* + L(c^*, d^*)] = TSC(c, d^*) - TSC(c^*, d^*) \geq 0$ (1)

$EC_2(c^*, d^*) = d^*$
$d < d^* \rightarrow EC_2(c^*, d) = d + L(c^*, d) - L(c^*, d^*)$
Therefore, $d < d^* \rightarrow EC_2(c^*, d) - EC_2(c^*, d^*) = [d + L(c^*, d)] - [d^* + L(c^*, d^*)] = [c^* + d + L(c^*, d)] - [c^* + d^* + L(c^*, d^*)] = TSC(c^*, d) - TSC(c^*, d^*) \geq 0$ (2)
$d > d^* \rightarrow EC_2(c^*, d) = d$
Therefore, $d > d^* \rightarrow EC_2(c^*, d) - EC_2(c^*, d^*) = d - d^* > 0$ (3)

(1)-(3) establish that (c^*, d^*) is a Nash equilibrium. □

Lemma 2 *Let negligence be defined as shortfall from due care. Let* $< C, D, \pi, H, d^*) >$ *be an application belonging to* $\mathcal{A}_{sf}$*. Then, under the incremental negligence rule we have:* $(\forall(\overline{c}, \overline{d}) \in C \times D)[(\overline{c}, \overline{d})$ *is a Nash equilibrium* $\rightarrow (\overline{c}, \overline{d}) \in M]$.

Proof: Let $< C, D, \pi, H, d^* > \in \mathcal{A}_{sf}$. Suppose $(\overline{c}, \overline{d})$ is a Nash equilibrium. Let c^* be such that $(c^*, d^*) \in M$.

$(\overline{c}, \overline{d})$ is a Nash equilibrium implies
$EC_1(\overline{c}, \overline{d}) \leq EC_1(c^*, \overline{d})$ (1)
and
$EC_2(\overline{c}, \overline{d}) \leq EC_2(\overline{c}, d^*)$ (2)
(1) $\wedge$ (2) $\rightarrow \overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \leq EC_1(c^*, \overline{d}) + EC_2(\overline{c}, d^*)$ (3)

Now,

$$\begin{aligned} EC_1(c^*, \overline{d}) &= c^* + L(c^*, d^*) && \text{if } d < d^* \\ &= c^* + L(c^*, \overline{d}) \leq c^* + L(c^*, d^*) && \text{if } d \geq d^* \end{aligned}$$

Thus, $EC_1(c^*, \overline{d}) \leq c^* + L(c^*, d^*)$ (4)

$EC_2(\overline{c}, d^*) = d^*$ (5)

(3)-(5) imply that:
$TSC(\overline{c}, \overline{d}) = \overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \leq EC_1(c^*, \overline{d}) + EC_2(\overline{c}, d^*) \leq c^* + d^* + L(c^*, d^*) = TSC(c^*, d^*)$ (6)

As total social costs are minimized at (c^*, d^*), it follows that $TSC(\overline{c}, \overline{d}) = TSC(c^*, d^*)$; and consequently we must have $(\overline{c}, \overline{d}) \in M$. The proposition therefore stands established. □

Proposition 2 *Let negligence be defined as shortfall from due care. Then, the incremental negligence rule is efficient for every application belonging to* $\mathcal{A}_{sf}$.

Proof: Follows immediately from Lemmas 1 and 2. □

Efficiency of negligence rule (i-sf) for the unilateral case was established by Kahan (1989). Proposition 2 generalizes Kahan's result by establishing efficiency of negligence rule (i-sf) for unilateral case as well as for bilateral case.

4 Negligence as Cost-Justified Untaken Precaution and Efficiency of Incremental Negligence Rule

If negligence is defined as existence of a cost-justified untaken precaution then the incremental version of the negligence rule is not efficient for every possible application as the following proposition shows.[3]

Proposition 3 *If negligence is defined as existence of a cost-justified untaken precaution then the incremental negligence rule is not efficient for every application belonging to* $\mathcal{A}_{up}$.

Proof Consider the following application belonging to $\mathcal{A}_{up}$.
Let $C = \{0, p_0c_0, c_0\}; D = \{0, d_0, \frac{d_0}{q_0}\}$; and $L(c,d), (c,d) \in C \times D$, be as given in the

[3]It can also be shown that if negligence is defined as existence of a cost-justified untaken precaution then the standard version of the negligence rule is not efficient for every possible application. See Jain (2006).

following array:

		d		
		0	d_0	$\frac{d_0}{q_0}$
	0	$c_0+\epsilon_1+\frac{d_0}{q_0}+\epsilon_2$	$c_0+\epsilon_1+(\frac{1}{q_0}-1)d_0$	$c_0+\epsilon_1-\epsilon_3$
c	p_0c_0	$(1-p_0)c_0+\epsilon_1+\frac{d_0}{q_0}+\epsilon_2$	$(1-p_0)c_0+\epsilon_1+(\frac{1}{q_0}-1)d_0$	$(1-p_0)c_0+\epsilon_1-\epsilon_3$
	c_0	$\frac{d_0}{q_0}+\epsilon_2$	$(\frac{1}{q_0}-1)d_0$	ϵ_4

where $p_0, q_0, c_0, d_0, \epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4 > 0$ are such that:[4]
(i) $p_0 < 1$ and $q_0 < 1$
(ii) $\epsilon_3 < \epsilon_1$
(iii) $\epsilon_4 < min\{\epsilon_2, (\epsilon_1 - \epsilon_3)\}$
(iv) $\epsilon_1 < (\frac{1}{q_0} - 1)d_0$.

(c_0, d_0) is the unique total social cost minimizing configuration.

We obtain $D^u(c,d)$, $(c,d) \in C \times D$, as given in the following array:

		d		
		0	d_0	$\frac{d_0}{q_0}$
	0	$\{d_0, \frac{d_0}{q_0}\}$	$\{\frac{d_0}{q_0}\}$	$\emptyset$
c	p_0c_0	$\{d_0, \frac{d_0}{q_0}\}$	$\{\frac{d_0}{q_0}\}$	$\emptyset$
	c_0	$\{d_0, \frac{d_0}{q_0}\}$	$\emptyset$	$\emptyset$

Therefore, injurer is negligent or nonnegligent at $(c,d) \in C \times D$, as given in the following array:

		d		
		0	d_0	$\frac{d_0}{q_0}$
	0	negligent	negligent	nonnegligent
c	p_0c_0	negligent	negligent	nonnegligent
	c_0	negligent	nonnegligent	nonnegligent

Now, expected costs of the victim at $(c_0, d_0) = EC_1(c_0, d_0)$
$= c_0 + L(c_0, d_0)$
$= c_0 + (\frac{1}{q_0} - 1)d_0$
$EC_1(p_0c_0, d_0)$

[4] $p_0 = q_0 = \frac{1}{2}, c_0 = d_0 = 10, \epsilon_1 = \epsilon_2 = 3, \epsilon_3 = \epsilon_4 = 1$ satisfy (i)-(iii).

$= p_0c_0 + L(p_0c_0, d_0) - \hat{L}_2(p_0c_0, d_0)$
$= p_0c_0 + [(1 - p_0)c_0 + \epsilon_1 + (\frac{1}{q_0} - 1)d_0] - [(\frac{1}{q_0} - 1)d_0 + \epsilon_3]$
$= c_0 + \epsilon_1 - \epsilon_3$
$EC_1(c_0, d_0) - EC_1(p_0c_0, d_0)$
$[(\frac{1}{q_0} - 1)d_0 - \epsilon_1] + \epsilon_3$
$> 0.$

Therefore it follows that (c_0, d_0) is not a Nash equilibrium. Consequently negligence rule (i-up) is not efficient for every application belonging to $\mathcal{A}_{up}$. □

5 Unilateral Case

Efficiency with respect to a class of applications $\mathcal{A}$ means efficiency with respect to every application belonging to $\mathcal{A}$. Therefore efficiency with respect to $\mathcal{A}$ implies efficiency with respect to every nonempty subset of $\mathcal{A}$ as well. As both standard and incremental versions of negligence rule are efficient for every possible application when negligence is defined as shortfall from due care, it follows that both (s-sf) and (i-sf) versions of negligence rule are efficient when care is unilateral rather than bilateral.

While efficiency with respect to a class of applications $\mathcal{A}$ implies efficiency with respect to every nonempty subset of $\mathcal{A}$; inefficiency with respect to $\mathcal{A}$ does not imply inefficiency with respect to every nonempty subset of $\mathcal{A}$. Inefficiency with respect to $\mathcal{A}$ merely entails the existence of at least one application belonging to $\mathcal{A}$ for which the rule is inefficient. Thus, it follows that a rule inefficient with respect to $\mathcal{A}$ may very well be efficient with respect to some proper subset of $\mathcal{A}$.

When negligence is defined as existence of an untaken precaution, the incremental version of negligence rule is inefficient with respect to $\mathcal{A}_{up}$ as seen above. The class $\mathcal{A}_{up}$ includes all applications with bilateral care as well as with unilateral care. It is of considerable interest to determine whether negligence rule (i-up) is efficient when care is unilateral. The next proposition shows that negligence rule (i-up) is inefficient even when care is unilateral.

Proposition 4 *If negligence is defined as existence of a cost-justified untaken precaution then the incremental negligence rule is not efficient for every application belonging to* $\mathcal{A}_{up}^{o_1}$.

Proof Consider the following application belonging to $\mathcal{A}_{up}^{o_1}$.
Let $C = \{0, \epsilon_1\}; D = \{0, d_0, \frac{d_0}{q_0}\}$; and $L(c, d)$, $(c, d) \in C \times D$, be as given in the following

array:

			d	
		0	d_0	$\frac{d_0}{q_0}$
c	0	$\frac{d_0}{q_0}+\theta+L_0$	$L_0+(\frac{1}{q_0}-1)d_0$	$L_0+\epsilon_2$
	ϵ_1	$\frac{d_0}{q_0}+\theta+L_0-\epsilon_3$	$L_0+(\frac{1}{q_0}-1)d_0-\epsilon_3$	$L_0+\epsilon_2-\epsilon_3-\epsilon_4$

where $q_0, d_0, L_0, \theta, \epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4 > 0$ are such that:[5]

(i) $q_0 < 1$
(ii) $\epsilon_3 + \epsilon_4 < \epsilon_1$
(iii) $\epsilon_2 < min\ \{\theta, \epsilon_4\}$
(iv) $\epsilon_1 < L_0$.
(v) $\epsilon_1 + \epsilon_2 < (\frac{1}{q_0} - 1)d_0$.

$(0, d_0)$ is the unique total social cost minimizing configuration.

We obtain $D^u(c, d)$, $(c, d) \in C \times D$, as given in the following array:

			d	
		0	d_0	$\frac{d_0}{q_0}$
c	0	$\{d_0, \frac{d_0}{q_0}\}$	$\emptyset$	$\emptyset$
	ϵ_1	$\{d_0, \frac{d_0}{q_0}\}$	$\{\frac{d_0}{q_0}\}$	$\emptyset$

Therefore, injurer is negligent or nonnegligent at $(c, d) \in C \times D$, as given in the following array:

			d	
		0	d_0	$\frac{d_0}{q_0}$
c	0	negligent	nonnegligent	nonnegligent
	ϵ_1	negligent	negligent	nonnegligent

Now, expected costs of the victim at $(0, d_0) = EC_1(0, d_0)$

[5] $q_0 = \frac{1}{2}, d_0 = 10, L_0 = 5, \theta = 4, \epsilon_1 = 4, \epsilon_2 = 1, \epsilon_3 = 1, \epsilon_4 = 2$ satisfy (i)-(v).

$= 0 + L(0, d_0)$
$= L_0 + (\frac{1}{q_0} - 1)d_0$

$EC_1(\epsilon_1, d_0)$
$= \epsilon_1 + L(\epsilon_1, d_0) - \hat{L}_2(\epsilon_1, d_0)$
$= \epsilon_1 + L(\epsilon_1, d_0) - [L(\epsilon_1, d_0) - L(\epsilon_1, \frac{d_0}{q_0})]$
$= \epsilon_1 + L_0 + \epsilon_2 - \epsilon_3 - \epsilon_4$

$$EC_1(0, d_0) - EC_1(\epsilon_1, d_0) = (\frac{1}{q_0} - 1)d_0 - \epsilon_1 - \epsilon_2 + \epsilon_3 + \epsilon_4 > 0.$$

This establishes that $(0, d_0)$ is not a Nash equilibrium. Consequently incremental negligence rule with negligence defined as existence of a cost-justified precaution is not an efficient liability rule for every application belonging to $\mathcal{A}^{o_1}$. □

The main reason why incremental negligence rule is not efficient even in the unilateral case when negligence is defined in terms of existence of cost-justified untaken precautions is that this way of defining negligence introduces strategic manipulability in the system. When negligence is defined in terms of existence of cost-justified untaken precautions, whether one is negligent or not depends not only on one's own care level but also on the care level of the other party. Consequently, the possibility of victim being in a position, by taking care which is socially deficient or socially excessive, to render an injurer who is taking socially optimal level of care negligent cannot be ruled out. The application which was considered to establish the preceding proposition was such that when both injurer and victim are taking socially optimal amounts of care, d_0 and 0 respectively, injurer is nonnegligent. However, by taking from a social perspective excessive care the victim can make injurer with care level d_0 negligent and thereby benefit himself by bringing about a reduction in his expected costs.

Depending on the application, the manipulation of a situation by victim can be done by taking a socially deficient level of precaution or by taking a socially excessive level of precaution. However, if one is considering only applications belonging to $\mathcal{A}_{up}^{o_1}$ then of course manipulation by victim by taking a socially deficient level of care is not possible; the only way victim can manipulate is by taking a socially excessive level of care. If a situation is such that victim can manipulate it by taking a socially excessive level of care then it must be the case there is some complementarity in victim's and injurer's precautions. In the application which was considered in the context of the preceding proposition there is a unique TSC minimizing configuration with victim taking 0 care and injurer d_0 care. Given that victim is taking 0 care if injurer increases his care from d_0 to $\frac{d_0}{q_0}$ reduction in expected loss is less than the increase of $(\frac{1}{q_0} - 1)d_0$ in care. However, if victim is taking $\epsilon_1 > 0$ care then injurer's increasing care from d_0 to $\frac{d_0}{q_0}$ results in a decrease in expected loss greater than the increase in care.

Thus, it seems that if one is considering an application belonging to $\mathcal{A}_{up}^{o_1}$ in which this kind of complementarities are not there then victim would not be able to manipulate and the source of inefficiency would no longer be there. While it is true that if applications belonging to $\mathcal{A}_{up}^{o_1}$ are suitably restricted to rule out complementarities then manipulation by victim would no longer be possible, efficiency is still not guaranteed as is shown in Proposition 6. The idea of absence of complementarities in victim's and injurer's precautions can be formalized as follows:

Let an application be called S-restricted iff $(\forall c, c' \in C)(\forall d, d' \in D)[c > c' \wedge d > d' \rightarrow [L(c', d) - L(c, d) \leq L(c', d') - L(c, d')] \wedge [L(c, d') - L(c, d) \leq L(c', d') - L(c', d)]]$. Let the subset of all S-restricted applications belonging to $\mathcal{A}_{up}$ be denoted by $\mathcal{A}_{up}^{S}$.

Remark 1 *$\mathcal{A}_{up}^{S}$ is the set of applications which do not exhibit complementarities in precautions by victim and injurer. It should be noted that the application which was considered for establishing Proposition 3 was S-restricted. Therefore the following stronger result also holds from which Proposition 3 can be derived as a corollary.*

Proposition 5 *If negligence is defined as existence of a cost-justified untaken precaution then the incremental negligence rule is not efficient for every application belonging to $\mathcal{A}_{up}^{S}$.*

Now we state and prove the proposition which says that even when care is unilateral and there are no complementarities in precautions by victim and injurer efficiency under the incremental negligence rule is not guaranteed.

Proposition 6 *If negligence is defined as existence of a cost-justified untaken precaution then the incremental negligence rule is not efficient for every application belonging to $\mathcal{A}_{up}^{o_1} \cap \mathcal{A}_{up}^{S}$.*

Proof: Consider the following application belonging to $\mathcal{A}_{up}^{o_1} \cap \mathcal{A}^{S}$.
Let $C = \{0\}; D = [0, 2\epsilon], \epsilon > 0$; and

$$\begin{aligned} L(0,d) &= 2\epsilon + \theta, \quad 0 \leq d \leq \epsilon \\ &= 2\epsilon - d, \quad \epsilon < d \leq 2\epsilon; \end{aligned}$$

where $\theta > 0$.

We obtain:
$(\forall d \in [0,\epsilon])[D^u(0,d) = (\epsilon, 2\epsilon]] \wedge (\forall d \in (\epsilon, 2\epsilon])[D^u(0,d) = \emptyset]$;
and consequently injurer is negligent at every $d \in [0,\epsilon]$ and nonnegligent at every $d \in (\epsilon, 2\epsilon]$.
Now, for $0 \leq d \leq \epsilon$,
$EC_2(0,d) = d + \hat{L}_2(0,d)$
$= d + 2\epsilon + \theta > 2\epsilon$
Thus, for $0 \leq d \leq \epsilon$ we have: $EC_2(0,d) > 2\epsilon$ (1)
For $\epsilon < d \leq 2\epsilon$,
$EC_2(0,d) = d \leq 2\epsilon$ (2)

From (1) and (2) it follows that no $(0,d), 0 \leq d \leq \epsilon$, can be a Nash equilibrium. From (2), it follows that no $(0,d), \epsilon < d \leq 2\epsilon$, can be a Nash equilibrium. Thus there does not exist any $(0,d) \in \{0\} \times D$ which is a Nash equilibrium; establishing the proposition. □

When care is unilateral and applications are S-restricted, strategic considerations introduced by the idea of negligence as existence of a cost-justified precaution get eliminated so that victim is not in a position to manipulate and bring about inefficiency. The reason why inefficiency can still be there is due to another problem which appears because of the way the notion of negligence is defined. As there is no specified level of care which injurer has to take in order to be nonnegligent, a rational injurer would like to choose the minimum amount of care consistent with being nonnegligent. This, however, may not be possible in some situations where set D_m has no minimum. This problem does not arise when negligence is defined as shortfall from due care regardless of whether $min\ D_m$ exists or not because of fixity of due care level. If we consider only those applications belonging to $\mathcal{A}^{o_1}_{up} \cap \mathcal{A}^{S}_{up}$ which are such that $min\ D_m$ exists then it can be shown that incremental negligence rule is efficient for these applications as is done in the proposition which follows.

Proposition 7 *If negligence is defined as existence of a cost-justified untaken precaution then the incremental negligence rule is efficient for every application belonging to* $\mathcal{A}^{o_1}_{up} \cap \mathcal{A}^{S}_{up} \cap \mathcal{A}^{m_2}_{up}$.

Proof: Consider any application $< C, D, \pi, H >$ belonging to $\mathcal{A}^{o_1}_{up} \cap \mathcal{A}^{S}_{up} \cap \mathcal{A}^{m_2}_{up}$. Let $min\ D_M = d_m$.

Consider any $(0,d) \in C \times D,\ d < d_m$.
As $(0,d) \notin M$ and $(0,d_m) \in M$, it follows that $d + L(0,d) > d_m + L(0,d_m)$ implying $L(0,d) - L(0,d_m) > d_m - d$. Which in turn implies that injurer is negligent at $(0,d)$.

Therefore,
$EC_2(0,d) = d + \hat{L}_2(0,d)$
$\geq d + L(0,d) - L(0,d_m)$, as $d_m \in D^u(0,d)$
$> d_m$. (1)
As $(0,d_m) \in M$, it follows that injurer is nonnegligent at $(0,d_m)$. Therefore his expected costs at $(0,d_m)$ are d_m. (2)
From (1) and (2) it follows that: given that victim takes care = 0, for injurer d_m is better than any $d < d_m$. (3)

Next consider any $(0,d) \in C \times D, d > d_m$.
$$\begin{aligned} EC_2(0,d) &= d + \hat{L}_2(0,d) && \text{if injurer is negligent at}(0,d) \\ &= d && \text{if injurer is nonnegligent at}(0,d) \end{aligned}$$
Thus injurer's expected costs at $(0,d)$ would be greater than d_m regardless of whether he is negligent or nonnegligent at $(0,d)$. As injurer's expected costs at $(0,d_m)$ are d_m, it follows that:
given that victim takes care = 0, for injurer d_m is better than any $d > d_m$. (4)

Next consider $(c,d_m) \in C \times D, c > 0$.
As the application under consideration belongs to $\mathcal{A}^S$, it follows that for any $d > d_m$:
$L(c,d_m) - L(c,d) \leq L(0,d_m) - L(0,d)$.
As $L(0,d_m) - L(0,d) \leq d - d_m$ in view of the fact that $(0,d) \in M$, it follows that we have:
$L(c,d_m) - L(c,d) \leq d - d_m$
which implies that injurer is nonnegligent at $(c,d_m), c > 0$.
Consequently, $EC_1(c,d_m) - EC_1(0,d_m) = [c + L(c,d_m)] - [0 + L(0,d_m)] = [c + d_m + L(c,d_m)] - [0 + d_m + L(0,d_m)] = TSC(c,d_m) - TSC(0,d_m) > 0$, as $(c,d_m) \notin M$ and $(0,d_m) \in M$.
Therefore we obtain:
given that injurer takes care = d_m, for victim 0 is better than any $c > 0$. (5)

(3)-(5) establish that $(0,d_m)$ is a Nash equilibrium. (6)

Suppose $(\overline{c},\overline{d}) \in C \times D$ is a Nash equilibrium.
$(\overline{c},\overline{d})$ being a Nash equilibrium implies:
$EC_1(\overline{c},\overline{d}) \leq EC_1(0,\overline{d})$; and (7)
$EC_2(\overline{c},\overline{d}) \leq EC_2(\overline{c},d_m)$ (8)
(7) and (8) imply that:
$EC_1(\overline{c},\overline{d}) + EC_2(\overline{c},\overline{d}) = \overline{c} + \overline{d} + L(\overline{c},\overline{d}) \leq EC_1(0,\overline{d}) + EC_2(\overline{c},d_m)$. (9)

First consider $\overline{d} < d_m$.

Injurer is negligent at $(0, \overline{d})$. Therefore: $EC_1(0, \overline{d}) = L(0, \overline{d}) - \hat{L}_2(0, \overline{d})$
$\hat{L}_2(0, \overline{d}) \geq L(0, \overline{d}) - L(0, d_m)$, as $d_m \in D^u(0, \overline{d})$ in view of the facts that $(0, \overline{d}) \notin M$ and $(0, d_m) \in M$
Therefore: $\overline{d} < d_m \rightarrow EC_1(0, \overline{d}) \leq L(0, d_m)$. (10)

Next consider $\overline{d} > d_m$
If at $(0, \overline{d})$ injurer is nonnegligent then: $EC_1(0, \overline{d}) = L(0, \overline{d})$
If at $(0, \overline{d})$ injurer is negligent then: $EC_1(0, \overline{d}) = L(0, \overline{d}) - \hat{L}_2(0, \overline{d})$
Thus: $EC_1(0, \overline{d}) \leq L(0, \overline{d}) \leq L(0, d_m)$
Therefore: $\overline{d} > d_m \rightarrow EC_1(0, \overline{d}) \leq L(0, d_m)$. (11)

(10) and (11) establish that: $EC_1(0, \overline{d}) \leq L(0, d_m)$. (12)

As has been seen above, injurer is nonnegligent at $(\overline{c}, d_m), c \geq 0$.
Therefore: $EC_2(\overline{c}, d_m) = d_m$. (13)

(9), (12) and (13) imply:

$$EC_1(\overline{c}, \overline{d}) + EC_2(\overline{c}, \overline{d}) = \overline{c} + \overline{d} + L(\overline{c}, \overline{d}) \leq EC_1(0, \overline{d}) + EC_2(\overline{c}, d_m) \leq L(0, d_m) + d_m. \quad (14)$$
$$(14) \rightarrow (\overline{c}, \overline{d}) \in M. \quad (15)$$

(15) establishes that all $(\overline{c}, \overline{d}) \in C \times D$ which are Nash equilibria are total social cost minimizing; and (6) establishes that there is at least one $(c, d) \in C \times D$ which is a Nash equilibrium.
This establishes the efficiency of incremental negligence rule with respect to $\mathcal{A}_{up}^{o_1} \cap \mathcal{A}_{up}^{S} \cap \mathcal{A}_{up}^{m_2}$. □

6 Concluding Remarks

The normative implications of the results of this paper are quite straightforward. Given the choice of negligence rule, from efficiency perspective it does not matter whether one opts for the standard rule or the incremental rule as long as the notion of negligence is defined as shortfall from due care level. The idea of defining negligence as existence of a cost-justified untaken precaution, on the other hand, is inconsistent with the objective of having an efficient version of the negligence rule for liability apportionment.

It is, however, the implications of the results of this paper for the positive analysis which are much more important. There are two important points which have emerged from Grady's meticulous analysis of how courts actually make use of negligence rule. One relates to the way courts view the idea of negligence. From Grady's analysis of cases it seems that in all likelihood courts use the notion of negligence in the cost-benefit sense, i.e., consider conduct negligent if there exists a cost-justified untaken precaution, but not otherwise. The second point relates to how courts determine the quantum of liability in cases of negligence. Once an injurer's conduct is found negligent on the ground that he did not take a particular cost-justified precaution, then the quantum of loss which can be attributed to the negligent conduct gets determined in a natural way as the amount of loss which would have been prevented by the untaken precaution in question. To hold the negligent injurer liable for the loss which he could have prevented seems particulary appropriate in such a context. The notion of negligence as existence of a cost-justified untaken precaution seems to be tied with the use of the incremental version of negligence rule. Thus taking Grady's two points together amounts to saying that courts by and large use the incremental version of the negligence rule with negligence defined as existence of a cost-justified untaken precaution. Even on theoretical grounds, one should not expect that courts in a non-inquisitorial system would try to find out all relevant information so as to determine the due care level correctly. Determination of negligence in terms of cost-benefit analysis of untaken precautions seems more in tune with an adversarial legal system. But, if the courts are using the incremental version with negligence defined in terms of cost-justified precautions then it could not be the case that court decisions are by and large efficient; with obvious and negative implications for the hypothesis that on the whole court decisions can be explained as if courts decide cases so as to bring about efficient outcomes.

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